PVPP'S

College Of Engineering

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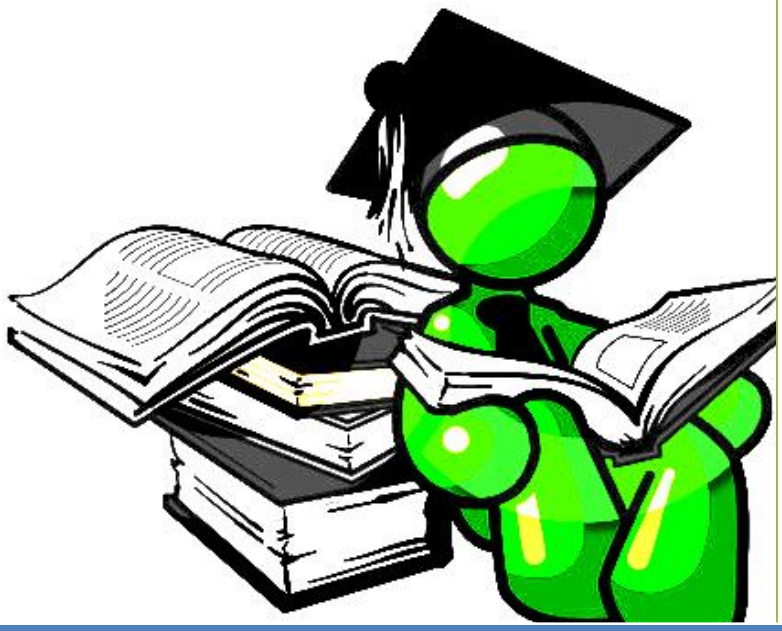
SH-2015



ACADEMIC BOOK

PADMABHUSHAN VASANTDADA PATIL PRATISHTHAN'S
SINCERITY SACRIFICE
SYSTEM 1990
COLLEGE OF ENGINEERING
Empower The Knowledge

SEMESRTER III

SE-ELECTRONICS

ACADEMIC BOOK

INDEX

Rules and Regulations

College Timings:
The college timing is from 8:45 AM to 4:45 PM .The students must follow the college timing.

Academic calendar and Time table:
The details of academic curriculum and activities are mentioned in the academic book. The students are required to strictly follow the class Time table and academic calendar.

Attendance:
All students are hereby informed that attendance for lectures/practical/tutorials is compulsory. Mumbai University does not allow students to appear for examination if their attendance is less than 75%.But for the good academic performance of the students, the department expects 100 % attendance in theory and practical separately.

Defaulters:
Defaulters list will be displayed monthly. The defaulter students are required to bring their parents/guardians within four days after the display of defaulters list. If students remain defaulter consistently he/she has to face the consequences as laid by the Mumbai University.

Identity card:
Student must wear ID during college hours in the campus.

Mobile Phone:
Use of cell phone is strictly prohibited in the college premises.

Examination:
As per the university norms, there will be two term test i.e Mid Term test and End Term test in the semester which is an integral part of Internal Assessment for every subject. Both the examination will be based on 40 % and 70 % of theory syllabus respectively for each subject and will be conducted as per the dates mentioned in the academic calendar. Attendance for both internal examination IS COMPULSORY .As per the university norms, no retest will be conducted under any circumstances. Separate passing heads is compulsory for internal and external examination for individual subjects. If the student fails in any of the exam he/she has to reappear in the concerned subject after the declaration of the result.

Practicals/tutorials/Assignments:
The Student should compulsory bring their rough and fair journal for the concerned subject for every practical and tutorials and get it checked regularly. Failing to do so, they will not be allowed for the practical. The Assignments for every subject should be submitted on regular basis.

The student must abide by the above mentioned rules and regulations laid down by the department for their better and brighter future.

ACADEMIC CALENDER SH-2015

COMMENCEMENT OF SEMESTER			
Sr. No.	Date	Activity	Responsibility
1	June 06, 2015	Mini Project Orientation Seminar for TE	TE Project Coordinators
	June 30, 2015	Display of Timetable	Time Table Committee
2	July14, 2015	Commencement of Term Address of HODs/ faculty to the student with faculty introduction. Theory and lab period as per time table. (Small orientation lecture are to be organized on first day and course content with industry relevance to be illustrated for all classes. Rules regulations to be explained too.)	HODs / CAs and faculties Distribution of Academic Book to all students Semester wise I,III,V,VII
3	July 18, 2015	Ramzan-Id	
4	July 24, 2015	Final Mini Project Group Formation(TE)	TE Project Coordinator
5	July 24, 2015	Project approval seminar and Display of approved project :Title and Name of Guide	BE Project coordinator
6	As per department's academic Calendar	Lecture Series	As per departmental Academic Time Table
7	July 31, 2015	Project Approval Seminar (TE)	Project coordinator
9	August 06, 2015	Display of approved Mini project(TE)	TE Project Coordinator
10	August 11-14, 2015	Introduction & Initial Mini Project development (TE)	TE Project Coordinator
11	August 14, 2015	**Display of defaulter's list – I**	**Class Advisors/ HODs (Reports to be generated through MIS)**
12	August 14,2015	Fresher's Party	Student's Council & SE Students
13	August 15,2015	Independence Day	Celebrated in the college as per circular
14	August 17-21, 2015	Literature Survey	TE Project Coordinator
15	August 18,2015	Parsi New Year	
16	August 17th ,18th & 19th ,2015	Students Feedback 1	Sys Admin (Online feedback in coordination with the departments)
17	**August 24-26,2015**	**Mid Term Test**	**HODs, CAs**

18		BE Project Review – I	BE Project coordinator
19	August Last week 2015	Mini Project Review	TE Project Coordinator
20	September 1-4, 2015	Practical work of Mini Project activities	TE Project Coordinator
21	September 6,2015	Gopalkala	
22	September 7 to 11th ,2015	Parent Teachers Interaction Meeting	HODs/ CAs
23	September 8-18, 2015	Implementation of Mini Project	TE Project Coordinator
24	**September 14 , 2014**	**Display of defaulter's list – 2**	**Class Advisors/ HODs (Reports to be generated through MIS)**
25	**September 14 to 18th,2015**	On line Examination	Coordinators/ Sys Administrator / Subject Teacher
26	**September 15, 2015**	Felicitation to toppers (Engineers Day)	Principal and Student's Council
27	**September 17, 2015**	Shri Ganesh Sthapana	Principal and Student's Council
28	**September 18, 2015**	Project Review – II and Submission of softcopy of synopsis	BE Project coordinator
29	**September 21,2015**	ShriGanesh Visarjan	Principal and Student's Council
30	September 22-25, 2015	Results & Conclusion	TE Project Coordinator
31	**September 25 ,2015**	Bakari ID (ID UL ZUHA)	
32	**September 27,2015**	Anant Chaturdashi	
33	**September 28-30 , 2015**	Students Feedback 2	Sys Admin (Online feedback in coordination with departments)
34	September Last week	Mini Project review-II	TE Project Coordinator
35	**October 2,2015**	Mahatma Gandhi Jayanti	
36	**October 5-7,2015**	**End Term Test**	**HODs, CAs**
37	October 09, 2015	Final certification and submission of synopsis	BE Project coordinator
38	October 06, 2015	Project Diary & Final report submitted to guide for approval	TE Project Coordinator
39	October 10, 2015	Final submission duly approved by guide	TE Project Coordinator
40	**October 12,2015**	**Third Defaulter List**	**Class Advisors/ HODs (Reports to be generated through MIS)**
41	**October 12-23,2015**	**Remedial Classes**	Coordinators with HODs (For weaker students)

42	October 22,2015	Dasara	
43	October 19-23 ,2015	Final Certification and submission	HOD (As per University schedule)
44	October 24,2015	Moharam	
45	October 26,2015	Final defaulter list	HODs
46	October 26,2015	Term End	HODs (As per University Schedule)
47	October 26 to November 7,2015	Conduction of Oral and Practical Examinations	Faculties (As per University Schedule)
48	November 11,2015	Laxmi Pujan	
49	November 12, 2015	Balipratiprada	
50	After Term End	Vacation for faculties 1st Slot	Exam In –Charge (As per University Schedule)
51	November 25, 2015	Gurunanak Jayanti	
52	18th Nov.2015 to 19th Dec. 2015	Non Vacation Slot	(As per University schedule)
53	18th November,2015 onwards	University Theory Examination for all Semesters	Exam In-Charge (As per University schedule)
54	20th December-3rd Jan. 2016	Vacation for faculties 2nd Slot	Exam In –Charge (As per University Schedule)
55	24th December,2015	ID-E-MILAD	
56	25th December,2015	CHRISTMAS	
57	January 04,2016	Commencement of FH-2016	

Summary:

1) Total Working Weeks : 15
2) Total Working days (excluding Saturdays, Sundays & examination) : 86
3) Total Working Days for teaching : 73
4) Available Periods for teaching :

Perweek	@
3	42
4	56
5	70

5) Tests :

Mid Term, End Term for (FE/SE/TE/BE)	Online Test for (FE/SE/TE/BE)
02 Written test	01

Note:

a. Attendance is compulsory from first day onwards
b. Those students who will remain absent on first day of academic semester, are compulsorily required to bring letter along with parents and meet the Principal/ HOD for permission to attend the college.
c. In case of absence (even for a day or hour), students are required to submit letters from parents at the time of attending the college.

Dr.Rajendra R.Sawant
Principal

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SH-2015

OBJECT ORIENTED PROGRAMMING METHODOLOGY

PADMABHUSHAN VASANTDADA PATIL PRATISHTHAN'S
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Empower The Knowledge

Mrs.NILIMA ZADE

Subject Plan

GROUP NAME: SIGNALS AND SYSTEM

COURSE TITLE: Object Oriented programming methodology

COURSE CODE:EXL 304

SEM : IV(SH 2015)

PRE-REQUISITE: Structured Programming language or any programming language

OBJECTIVES :

1. To understand the concept of Object Oriented Programming
2. To help student to understand how to use a programming language such as JAVA to resolve problems.
3. To impart problems understanding, analyzing skills in order to formulate Algorithms.
4. To provide knowledge about JAVA fundamentals: data types, variables, keywords and control structures.
5. To understand methods, arrays, inheritance, Interface, package and multithreading.
6. To understand the concept of Applet.
7. To emphasize intuitive understanding and practical implementations of the theoretical concepts.
8. To develop an appreciation of the application of his/her knowledge in actual industry and project work.
9. To prepare the students to excel in post graduate studies.

OUTCOME :

1. Students will be able to code a program using JAVA constructs.
2. Given an algorithm a student will be able to formulate a program that correctly implements the algorithm.
3. Students will be able to generate different patterns and flows using control structures.
4. Students will be able to make use of recursion in their programs.
5. Students will be able to use thread methods, thread exceptions and thread priority.
6. Students will implement method overloading in their code.
7. Students will be able to demonstrate reusability with the help of inheritance.
8. Students will be able to make more efficient programs.

LEARNING RESOURCES: -

RECOMMENDED BOOKS:-

1. **'Head First Java ' by Kathy Sierra**
2. **Java, Abeginner's guide, 5th Edition Herbert Schildt**
3. **Core Java Volume I Fundamentals by Cay S. Horstmann**

COURSE MATERIALS MADE AVAILABLE

1. Course instructional objectives & outcomes
2. Syllabus
3. Chapterwise Question Bank

Evaluation :

Theory Exam	No
Internal assessment:-. The average marks of Mid-term test (20 M) & End-term test (20 M) will be considered as final IA marks	No
Practrical exam and Oral	50 M
Term Work	25 M
Total	75 M

List of Experiments

Atleast 10 experiments based on the entire syllabus

Expt.No.	Name of the Experiments
1	Implementation of JDK Environment and Tools
2	Implement a program to create a class and its instance.
3	Implementation of different operators in Java.
4	Implementation Different Loops and expressions in Java
5	Implementation of Different methods in Java
6	Implementation of arrays in Java.
7	Implementation of Inheritance in Java.
8	Implementation of Interface.
9	Creating Package.
10	Implementation of Threads.
11	Creating Applet.

Chapterwise Plan

Subject Title: Object Oriented programming methodology

Chapter No. : 1

Chapter Name : Fundamental concepts of object oriented programming

Approximate Time Needed : 04hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
1	Overview of Programming Introduction to the principles of object-oriented programming : Classes, Objects, Messages,
2	Abstraction, Encapsulation, Inheritance, Polymorphism
3	exception handling, and object-oriented containers
4	Differences and Similarity between C++ and JAVA

Objectives:

1. The Student will learn fundamental concepts of classes and objects.
2. Learn basic four pillars of OOP
3. Learn basic structure of simple java program.
4. Learn difference between structured and object oriented language.

Lesson Outcome:

Students will able to

1. Understand difference and similarities in C++ and Java
2. Implement first simple java programs

Model Questions:

1. **What do you mean by OOP**
2. **What do you mean by Abstraction , encapsulation, inheritance, polymorphism.**
3. **Compare Java with C++**

Chapterwise Plan

Subject Title: Object Oriented programming methodology

Chapter No. : 2

Chapter Name : Fundamental of Java Programming

Approximate Time Needed : 04hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
5	Features of Java JDK Environment & tools
6	Structure of java program Keywords , Data types, Variables, Operators, Expressions
7	Decision Making, Looping, Type Casting
8	Input output using scanner class

Objectives:

1. Learn how to install JDK environment and tools
2. Learn fundamental concepts of Java programming.
3. Learn control structure
4. Learn how to give input and get output in java programming

Lesson Outcomes:

The student will be able to

1. Understand basic features Java
2. Write java programs to implement operators, loops scanner class.

Model Questions:

1. Implement a program to calculate the area of circle
2. Implement a program to demonstrate the use of ternary operator
3. Implement a program using arithmetic operators to perform algebraic operations on two numbers
4. Implement a program to demonstrate the implementation of prefix and postfix increment operators.
5. Implement a program to demonstrate while loop, do while loop,for loop.

Chapterwise Plan

Subject Title: Object Oriented programming methodology

Chapter No. : 3

Chapter Name : Classes and Objects

Approximate Time Needed : 06hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
9	Creating Classes and objects, Memory allocation for objects
10	Passing parameters to Methods, Returning parameters
11	Method overloading,
12	Constructor and finalize( )
13	Arrays : Creating an array
14	Types of Array : One Dimensional arrays,Two Dimensional array

Objectives:

1. The Student will learn fundamental concepts and structure of classes,
2. Learn structure of method and types of methods
3. Learn concepts and structure of arrays

Lesson Outcome

After completion of this module student will be able to

1. Design code to implement classes, methods.
2. Design code to Implement arrays.

Model Questions:

1. Write a program to demonstrate use of a method, which return value

2. Write a program to calculate the area of square and rectangle by overloading the area method.

3. Write a program to create an array to store 10 integer values. Also initialize the array with 10 random numbers, sort the array, and display the array elements

4. Write a program to add two rectangular matrix.

Chapterwise Plan

Chapter No. : 4

Chapter Name :Inheritance , Interface and Package

Approximate Time Needed : 06hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
15	Types of Inheritance : Single ,Multilevel, Hierarchical
16	Method Overriding, Super keyword, Final Keyword,
17	Abstract Class
18	Interface
19	Interface
20	Packages

Objectives:

The Student will learn Inheritance, Abstract class, Interface, Packages.

Lesson Outcomes

1. Write code to implement inheritance, method overriding, interface..
2. Create package.

Model Questions:

1. What is overriding
2. Consider an employee class, which contains fields as name and desg, and a subclass,which contain a field sal. Write a program for inheriting this relation.
3. Write a program for displaying the use of the super and final keyword
4. Write a program to implement an abstract method .
5. Write a program to calculate the area by using an interface
6. Write a program to create package.

Chapterwise Plan

Subject Title: Object Oriented programming methodology

Chapter No. : 5

Chapter Name :Multithreading

Approximate Time Needed : 06 hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
21	Thread
22	Life cycle of thread
23	Methods
24	Priority in multithreading

Objectives:

The Student will learn Threads in java

Outcomes:

The students will be able to Write code to implement thread.

Model Questions:

1. Write a program to create and use thread
2. Write a program to demonstrate sleep() and join() method.
3. Write a program to create and use multiple threads

Chapterwise Plan

Subject Title: Object Oriented programming methodology
Chapter No. : 6 **Chapter Name : APPLET** **Approximate Time Needed : 02hrs**
Lesson Schedule :

Lecture No.	Portion covered per hour
25	Applet Life cycle
26	Creating applet, Applet tag

Objectives:

To make students learn about applet.

Lesson Outcomes:

The students will be able to create applet

Model Questions:

1. Write a program to demonstrate life cycle of an applet
2. Write a program to animate a ball.

Practicals (Java Programs)

Exercise 1

1. Implement java code to create class Area to find out area of rectangle. Create method to find area when integer parameter and overload the method for type double parameter.

2. Implement java code to create a class to find area of circle. Design getter, setter method and method to find area and constructor for the class. Create another class to design main method and call all the methods.

Exercise 2

1. Subtract 10 paisa from 1 crore Rs using float. Display the answer.
   - Repeat using double variable.
   - Repeat using BigDecimal class.

2. Create a java program that has 4 classes. Keep 2 classes in one package. Keep 1 class in second package. Keep the file that contains main function outside any package.

3. The bank offers the following types of accounts to its customers savings, current and money market account. Customers are allowed to deposit and withdraw money from an account and earn interest on the account. each account has interest rate.

-Write an application that will calculate the amount of interest earned for a bank account.

- Identify potential classes in this problem domain.
- Identify the responsibilities of the classes.

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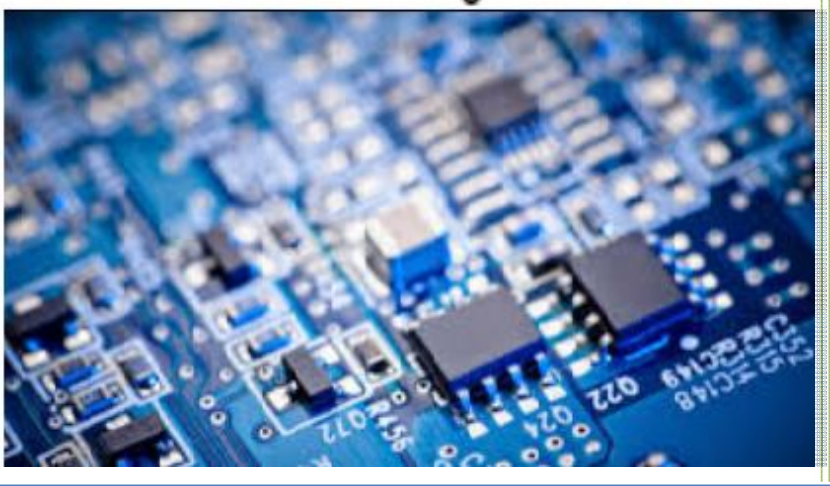
SH-2015

circuit theory



CIRCUIT THEORY

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Empower The Knowledge

Mrs. NILIMA ZADE

Subject Plan

GROUP NAME	**: Electrical**
COURSE TITLE	**:** Circuit Theory
COURSE CODE	**: EXC 304**
SEM	**: IV (SH 2015)**
PRE-REQUISITE	: **Basic Electrical and Electronics Engineering**

OBJECTIVES :

1. To analyze the circuits in time and frequency domain
2. To study network functions, inter relationship among various circuit parameters, solve complex network using these parameters.
3. To analyze and synthesize circuits and
4. To become familiar with the propagation of signals/wave through transmission lines.
5. To develop an appreciation of the application of his/her knowledge in actual industry and project work.
6. To prepare the students to excel in post graduate studies.

OUTCOME :

1. Through test and laboratory exercises, students will be able to apply their knowledge in solving complex circuits.
2. Students will be able to evaluate the time and frequency response which is useful in understanding behavior of electronic circuits and control system.
3. Student will be able to understand how the power or information in terms of electromagnetic energy is transmitted through the transmission lines and importance of impedance matching.
4. To prepare the students to excel in post graduate exams.

LEARNING RESOURCES: -

RECOMMENDED BOOKS:-

1. Franklin F Kuo, *"Network Analysis and Synthesis",* Wiley Toppan,

2. M E Van Valkenburg, *"Network Analysis",* Prentice-Hall of India Pvt Ltd, New Delhi

3. K V VMurty and M S Kamth*, "Basic Circuit Analysis",* Jaico Publishing house, London

4. A. Chakrabarti*, "Circuit Theory",* Dhanpat Rai and Co.,New Delhi

5. Reinhold Ludwig and Pavel Bretchko, "*RF Circuit Design"*, Pearson Education, Asia

6. Joseph J. Carr, "*Secrets of RF Circuit Design"*, Tata McGraw-Hill, New Delhi

COURSE MATERIALS MADE AVAILABLE

1. Course instructional objectives & outcomes
2. Syllabus
3. Chapterwise Question Bank

Evaluation :

Theory Exam	80 M
Internal assessment:-. The average marks of Mid-term test (20 M) & End-term test (20 M) will be considered as final IA marks	20 M
Oral	25 M
Term Work	25 M
Total	150 M

List of Experiments

Atleast 10 experiments based on the entire syllabus

Expt.No.	Name of the Experiments
1	To Perform Superposition Theorem.
2	To Perform Thevenin's Theorem.
3	To Perform Norton's Theorem.
4	To determine Z parameter of the two port network.
5	To determine H parameter of the two port network.
6	To perform the measurement of inductance, capacitance and resistance of given components by Using Q-meter.
7	To Study various type of sensors like RTD,Thermisters,Thermisters, Thermocouples-Their Ranges, And its Applications
8	To perform the measurement of frequency by using Lissajous figures using CRO
9	To perform the strain measurement using strain guage and determine it's linearity & sensitivity.
10	To perform the characteristics of LVDT and determine it's sensitivity.

Chapterwise Plan

Subject Title: Circuit Theory

Chapter No. : 1

Chapter Name : Circuit Analysis

Approximate Time Needed : 09 hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
1	Analysis of circuits without controlled sources using generalized loop
2	Analysis of circuits with controlled sources using generalized loop
3	Superposition Theorem
4	Thevenins Theorem
5	NortonsThm
6	MillmanThm
7	Analysis of coupled circuit
8	equivalent circuit and solution using loop analysis
9	Series and parallel resonance circuits

Objectives:

4. Student will learn about analysis of circuits and basic theorms
5. Learn effect of mutual inductance in coupled circuits
6. Learn resonance circuit.

Lesson Outcome:

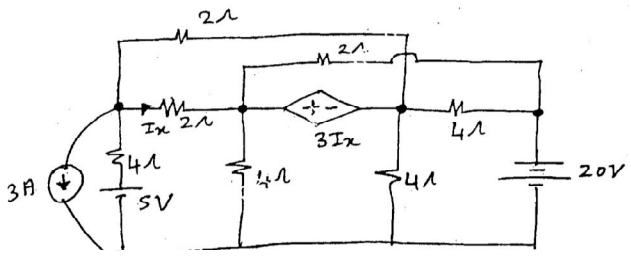
Students will able to

3. Through test and laboratory exercises, students will be able to apply their knowledge in solving complex circuits.
4. Apply knowledge in solving coupled circuits

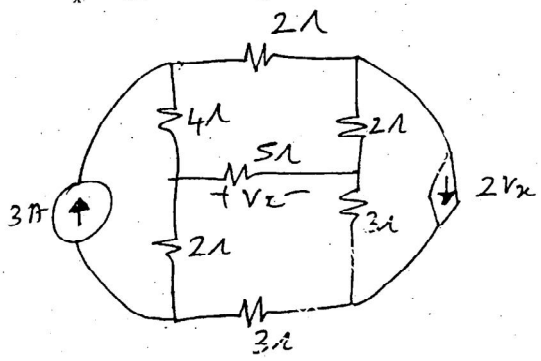
Model Questions:

JUNE 2014

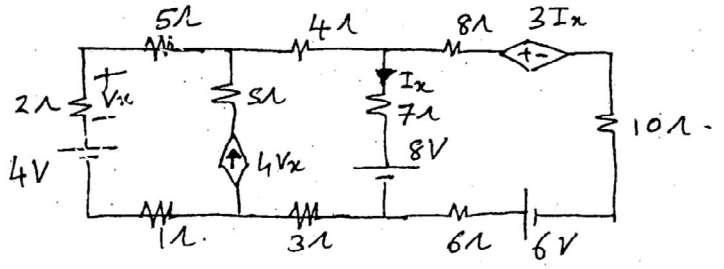
2. (a) Obtain pover supplied by dependent voltage source 10



(c) Obtain V_x using some shifting and source transformation technique. 6



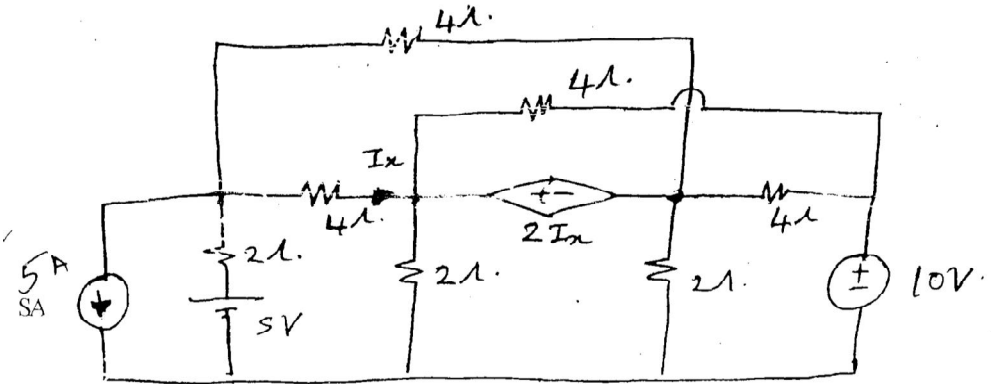
6. (a) Obtain Thevenin's equivalent circuit :- 8



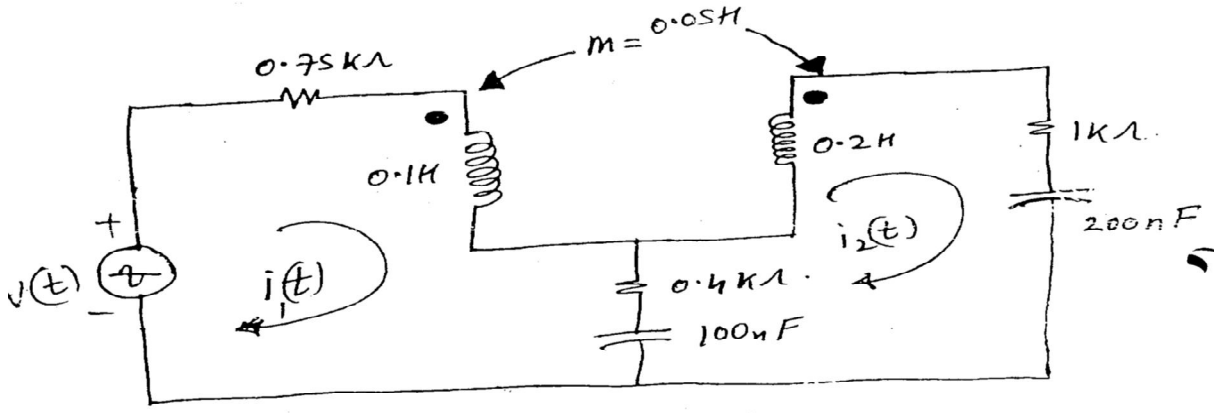
Hence find current flowing through 10Ω load.

DEC 2014

(b) Obtain Power associated with dependent voltage source by using Nodal analysis. 8

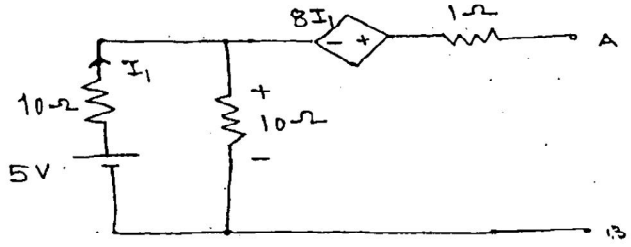


(b) Find current flowing in both coils. If applied input voltage is v (t)= 230 $\sqrt{2}$ sin [5000 t-30°]

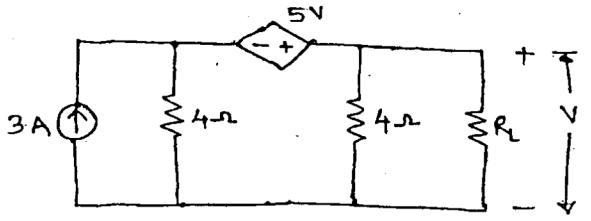


2.

(a) Find the thevnin's equivalent network for terminals A and B. **4**



2. (a) In the given network, what will be the R_L to get maximum power delivered to it. Calculate power. **8**



Chapterwise Plan

Subject Title: Circuit Theory

Chapter No. : 2

Chapter Name : Time and Frequency Domain Analysis

Approximate Time Needed : 12hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
10	Time domain analysis of R-L and R-C circuits: Forced and natural response,
11	time constant, initial and final values
12	Transient and steady state time response, solution using universal formula.
13	Transient and steady state time response, solution using universal formula.
14	Time domain analysis of R-L-C circuits: Forced and natural response, effect of damping.
15	Solution using second order equation for standard input signals: Transient and steady state time response
16	Frequency domain analysis of RLC circuits: S-domain representation,
17	applications of Laplace Transform in solving electrical networks, driving point and transfer function,
18	applications of Laplace Transform in solving electrical networks, driving point and transfer function,
19	applications of Laplace Transform in solving electrical networks, driving point and transfer function,
20	Poles and Zeros, calculation of residues by analytical
21	graphical method,.
22	frequency response

Objectives:

The Student will learn

1. To analyze the circuits in time and frequency domain
2. To study network functions, inter relationship among various circuit parameters, solve more complex network using these parameters.

Lesson Outcomes:

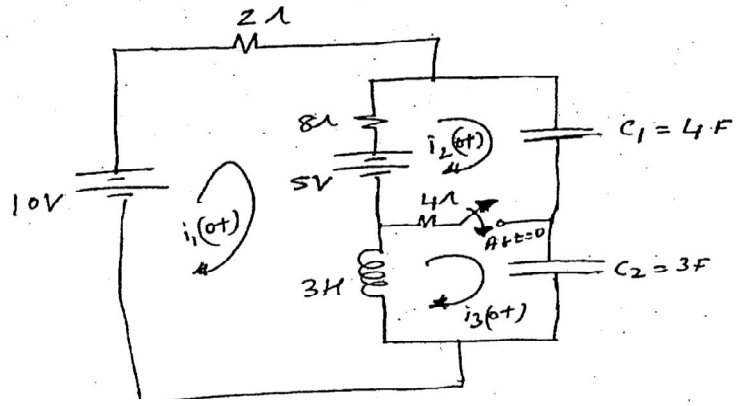
The student will be able to

1. students will be able to apply their knowledge in solving complex circuits.
2. Students will be able to evaluate the time and frequency response which is useful in understanding behavior of electronic circuits and control system.

3. Model Questions:

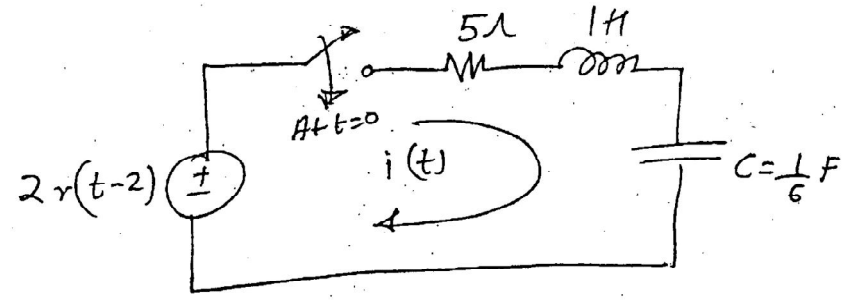
4. June 2014
5.

3. (a) Obtain $i_1(o+)$, $i_2(o+)$ and $i_3(o+)$ **10**



6.
7.

5. (a) Obtain i(t) for t > 0 **8**

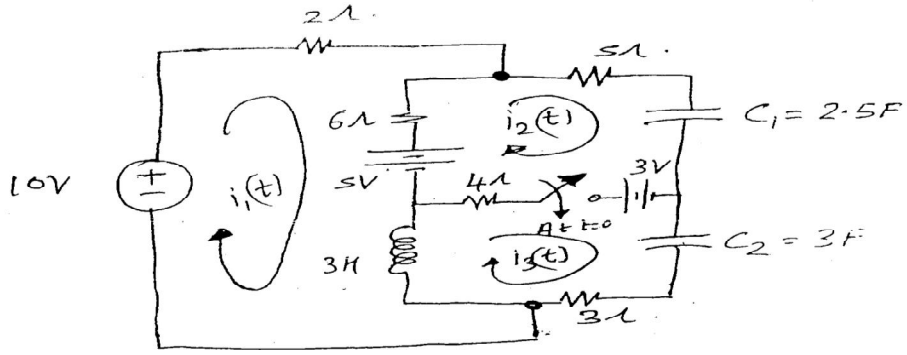


Where r(t) is a ramp signal.

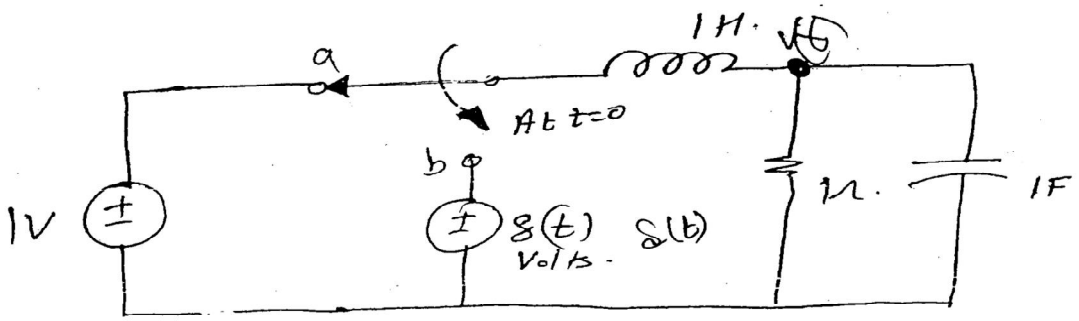
8.
9.
10. **Dec 2014**

11.

(b) Find $i_1(t)$, $i_2(t)$ and $i_3(t)$ at $t = 0^+$ 8

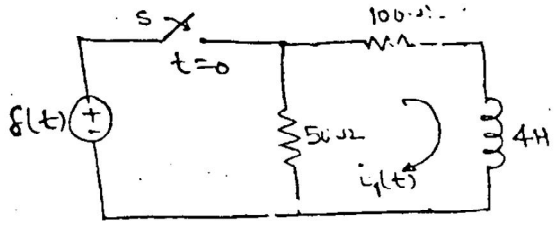


(b) Obatin v(t) for $t \geq o$ Use Laplace Transform method. 10

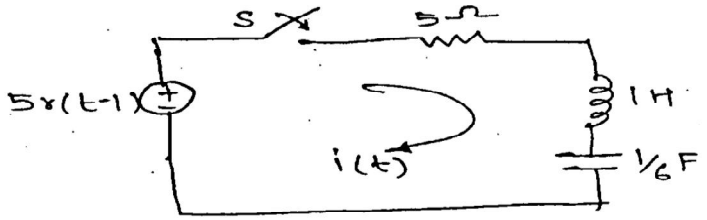


June 2015

(b) For the network shown, the switch is closed at t = 0. Find the current $i_1(t)$ for $t > 0$ 4



(b) For the network shown, determine the current i(t) when the switch is closed at t = 0 with zero initial conditions. **8**



Chapterwise Plan

Subject Title: Circuit Theory

Chapter No. : 3

Chapter Name : Synthesis of RLC Circuits

Approximate Time Needed : 08hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
23	Positive real functions: Concept of positive real function,
24	Testing for Hurwitz polynomials
25	Testing for Hurwitz polynomials
26	Testing for necessary and sufficient conditions for positive real functions
27	Testing for necessary and sufficient conditions for positive real functions
28	Synthesis of RC, RL, LC circuits: Concepts of synthesis of RC, RL, LC
29	Synthesis of RC, RL, LC circuits: Concepts of synthesis of RC, RL, LC
30	Driving point functions

Objectives:

1. Learn the positive real function and Hurwitz polynomial.
2. Learn the basic concepts synthesis of circuit
3. Understand driving point function.

Lesson Outcomes:

The student will be able to

12. apply their knowledge to synthesize the circuit.
13. Students will be able to evaluate positive real function and test Hurwitz polynomial

Model Questions:

June 2014

1. (a) Check for Hurwitz polynominal

$$Q(s) = s^5 + s^3 + s^1$$

$$Q(s) = s^4 + 6s^3 + 8s^2 + 10$$

(b) Check for p.r.f. test **6**

$$F(s) = \frac{2s^2 + 2s + 1}{s^3 + 2s^2 + s + 2}$$

(c) Compare Cauer Form I and Cauer Form II of a LC Network. **6**

$$Z(s) = \frac{2(s^2+1)(s^2+4)}{s(s^2+2)}$$

1.

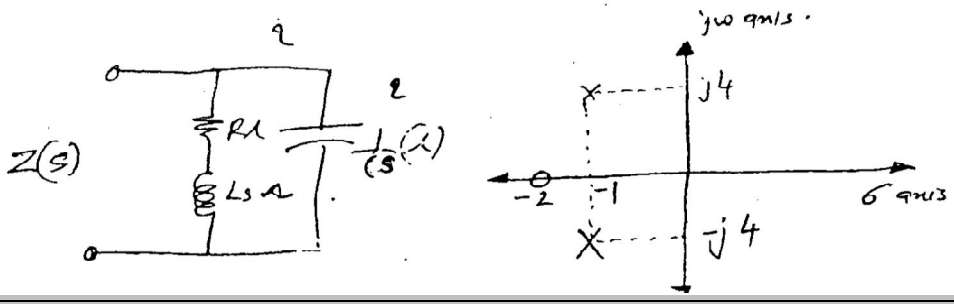
1. (a) Test for following polynominal using continued fraction expansion only

$$P(s) = s^6 + 2s^5 + 3s^4 + 4s^3 + 3s^2 + 2s + 1$$

(c) Compare Foster form I and Foster Form II of an LC N/W

$$z(s) = \frac{6s(s^2+4)}{(s^2+1)(s^2+64)}$$

(d) The ploe -zero diagram of driving point impedacne function is shown At d.c. the input inpedance is resistive and equal to 2 Ω Determine value of R, Land C.



Chapterwise Plan

Subject Title: Circuit Theory

Chapter No. : 4

Chapter Name :Two Port Networks

Approximate Time Needed : 08hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
31	Parameters: Open Circuit, Short Circuit,
32	Transmission and
33	Hybrid parameters,
34	Relationships among parameters,
35	Reciprocity and symmetry conditions.
36	Series/parallel connection:
37	T and Pi representations,
38	Interconnection of two port networks

Objectives:

1. Provide an understanding network parameters
2. Study the effect of interconnection of two port networks.

Lesson Outcomes

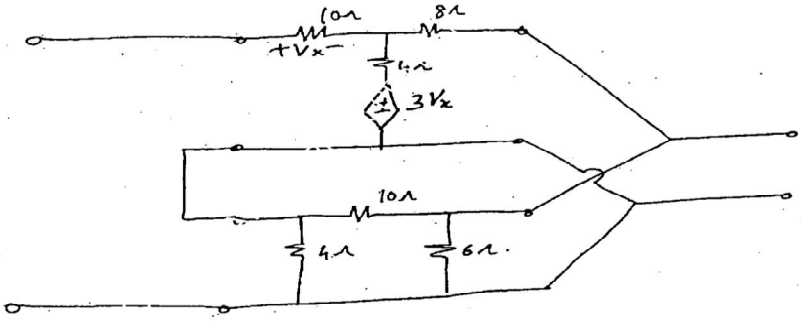
Student will be able to analyse

1. Analyze different network parameter
2. To identify different interconnection two port network and its effect;

Model Questions

(c) Obtain Transmission parameters in terms of 'z' parameters.

4. (a) Obtain hybrid parameters of the intercorrected 'Two' 2-port networks **8**



Chapterwise Plan

Subject Title: Digital Signal Processing and Processors

Chapter No. : 5

Chapter Name : Filters and attenuators

Approximate Time Needed : 08hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
39	Basic filter circuits: Low pass, high pass,
40	Band pass and band stop filters,
41	Transfer function, frequency response,
42	cutoff frequency, bandwidth, quality factor,
43	Attenuation constant, phase shift, characteristic impedance.
44	Concept of design and analysis of filters: Constant K, M derived and composite filters
45	Attenuators: Basic concepts, classification,
46	Attenuation in dB, K factor (impedance factor) and design concepts .

Objectives:

3. Student will learn Basic filters design and its analysis
4. Learn basic concepts of attenuators

Outcomes:

The students will

1. Understand filter design
2. Concepts of attenuators
3. Able to relate the knowledge for higher semester subjects

Model Questions:

(c) Explain various types of filters. 4

Chapterwise Plan

Subject Title: Circuit Theory

Chapter No. : 6

Chapter Name : Transmission Line

Approximate Time Needed : 09hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
47	Power frequency lines: Representation, losses and efficiency in power lines,
48	Effect of length, calculation of inductance and capacitance (numerical problems not expected)
49	Radio frequency lines: Representation,
50	propagation constant, attenuation constant,
51	phase constant, group velocity, input impedance, characteristic impedance,
52	Reflection coefficient, standing wave ratio,
53	VSWR, ISWR, S-parameters
54	Smith Chart: Impedance locus diagram, impedance matching
55	Smith Chart: Impedance locus diagram, impedance matching

Objectives:

To analyze and synthesize circuits and to become familiar with the propagation of signals/wavethrough transmission lines.

Lesson Outcomes:

Student will be able to understand how the power or information in terms of electromagnetic energy is transmitted through the transmission lines and importance of impedance matching.

Model Questions:

1. (b) Derive an expression for characteristic equation of a transmission line. Also obtain α, β and γ of the line. **6**

2. (c) The paranelex of a transmission line are G = 2.25 m Ω /km, R = 65 Ω /km, L = 1.6 m H/ km, C=.1 μ F/km find charterstic impedance and the propogation constant of the line at a frequency of 1 KHz.

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ELECTRONIC INSTRUMENTS & MEASUREMENTS

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Mr. ANAGHA DHAVALIKAR

Subject Plan

GROUP NAME : INSTRUMENTATION & CONTROL	
COURSE TITLE : Electronic Instruments and Measurements **COURSE CODE : S.E. Elex** **SEM : III**	**TOTAL TIME ALLOTTED:** **52 Hrs**
PRE-REQUISITE : System of units, Measuring Instruments.	

RATIONALE

The aim of the subject is to introduce the students to the concept of Measurement, Instrumentation and their application areas. Knowledge of different measuring instruments and transducers, helps the students to use them while designing and developing real world problems.

Course Objective:

1. In depth knowledge of measurement methods and instruments of electrical quantities.
2. Understanding design aspects and performance criterion for measuring instruments.
3. Implementation of the different signal generators and its analysis techniques.
4. To understand the working principle of the transducers.
5. To make the students aware about the advances in Instrumentation.

Course Outcomes:

1. An ability to apply knowledge of electronic instrumentation for measurement of electrical quantities
2. Ability to apply the principles and practices for instrument design and development to problems.
3. Ability to select and use the hardware for measurements and instrumentation.
4. An ability to design and conduct experiments for measurement and ability to analyze and interprets data.

COURSE MATERIALS MADE AVAILABLE

1. Course instructional objectives & outcomes
2. Syllabus
3. Chapter wise Question Bank

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Recommended Books:

1. H. Oliver and J. M. Cage, Electronic Measurement and Instrumentation, McGraw Hill, 3rd edition.
2. W. Cooper, A. Helfric, Electronic Instrumentation and Measurement Techniques, PHI, 4th edition.
3. C. S. Rangan, G.R. Sarma, V.S.V. Mani, Instrumentation Devices and Systems, Tata McGraw Hill, 9th edition.
4. A. K. Sawhney, Electrical & Electronic Instruments & Measurement, Dhanpat Rai and Sons, Eleventh ed., 2000.
5. Dally, William F. Riley and Kenneth G, Instrumentation for Engineering Measurements, James John Wiley and Sons. Inc., 2nd Edition 1993.
6. A.J. Bowens, Digital Instrumentation, McGraw-Hill, latest addition.
7. J.J.Carr, Elements of Electronic Instrumentation and Control, Prentice Hall, 3rd edition.

Evaluation :

University:

End Semester Examination: (80 Marks, 3 Hrs.)

1. Question paper will comprise of 6 questions, each carrying 20 marks.
2. The students need to solve total 4 questions.
3. Question No.1 will be compulsory and based on entire syllabus.
4. Remaining question (Q.2 to Q.6) will be set from all the modules.
5. Weight age of marks will be as per Blueprint.

Internal Assessment:

Test 1 – 20 marks

Test 2 – 20 marks

Chapterwise Plan

Subject Title: Electronic Instruments and Measurements

Chapter No. : 1

Chapter Name : Principles of Measurements

Approximate Time Needed : 6 hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
1	Subject Orientation.
2	Generalized Measurement System.
3	Applications of instrument systems.
4	Static characteristics of instruments.
5	Dynamic characteristics of instruments.
6	Errors in measurement.

Objectives: The student will learn,

1) An introduction to the concepts of measurement, essential to appreciate problems associated with instrumentation.

2) The basic characteristics, sources of error and the behavior of first and second order systems.

3) The performance characteristics of an instrumentation system is judged by how faithfully the system measures the desired input and how thoroughly it rejects the undesirable inputs. Quantitatively, it relates to the degree of approach to perfection.

4) The basic purpose of instrumentation in a process is to obtain the requisite information pertaining to the successful completion of the process.

Outcomes: The student should be able to,

1) Discuss system configuration.

2) Discuss response of system to standard inputs.

3) Discuss performance characteristics and error analysis of measuring system.

Model Questions:

Q.1 What are the various types of errors in measuring systems?

Q.2 What are the different errors encountered in measurements? Explain with suitable examples.

Q.3 Explain static and dynamic characteristics of measurements.

Q.4 Compare and contrast Reproducibility and Repeatability of an instrument.

Q.5 What are the characteristics of instrument? Explain any four characteristics.

Q.6 Explain generalized instrumentation system in short.

Chapterwise Plan

Subject Title: Electronic Instruments and Measurements

Chapter No. : 2

Chapter Name : Test & Measuring Instruments

Approximate Time Needed : 10hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
7	Analog Multi-meters.
8	Electronic Voltmeter.
9	FETs and op-amps.
10	Multi-meter specifications.
11	Wheatstone, Kelvin's and Mega Ohm Bridge.
12	Maxwell and Hey Bridge.
13	Schering Bridge.
14	Q-Meter.
15	Application of Q-meter.
16	DMM.

Objectives: The student will learn that

1. Analog and Digital Multi-meters.
2. The various methods use for resistance measurement.
3. The various bridges are use for inductance measurement.
4. The capacitance measurement by using bridge.
5. Q-meter for RLC measurement.

Outcomes: The student should be able to

1. Principle, Operation and specification of analog multi-meter.
2. Various conversion methods.

3. Principle, Operation and specification of digital multi-meter.
4. Resistance, inductance and capacitance measurement methods.
5. Kelvin's double bridge and Wheatstone bridge.
6. Maxwell's, Hay's bridge.
7. Schering bridge.

Model Questions

Q.1 What is Q meter? Explain anyone of the types of Q meter with the help of circuit diagram.

Q.2 Draw and explain Kelvin's Bridge.

Q.3 Draw and explain anyone of the types of electronic voltmeter. State its two advantages over analog voltmeter.

Q.4 Explain Schering bridge for measurement of capacitance. Derive the equation of unknown capacitance at balanced condition.

Q.5 What are the advantages of electronic voltmeter over the other voltmeters?

Q.6 Explain the applications and the limitations of the Wheatstone bridge.

Q.7 Explain the various performance parameters of digital voltmeter.

Q.8 Explain multi-range ohmmeter with diagram.

Q.9 Draw and explain FET as a voltmeter. What are its sensitivity considerations?

Q.10 How will you find the value of capacitance with the help of Schering bridge? Explain with the help of derivation and vector diagram.

Q.11 Compare a true rms meter with an average responding meter.

Q.12 State general characteristics of digital voltmeter. Explain with block diagram successive approximation type of DVM.

Q.13 Draw a neat circuit diagram and explain the working of an analog electronic voltmeter using FET bridge.

Q.14 What are the factors involved in the selection of electronic analog voltmeter?

Q.15 How is Kelvin Double Bridge different from Wheatstone bridge? Explain.

Q.16 Explain inductance measurement by using Maxwell and Hey bridge.

Chapterwise Plan

Subject Title: Electronic Instruments and Measurements

Chapter No. : 3

Chapter Name : Oscilloscopes

Approximate Time Needed : 10hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
17	Block diagram of CRO
18	Front panel of CRO.
19	Control and sweep modes of CRO.
20	Circuit diagram of time base, triggered time base,
21	Delayed time base, external triggering circuit.
22	Single and dual beam CRO.
23	Measurement of voltage, time and frequency on CRO.
24	Lissajous figures.
25	Block diagram and front panel of DSO.
26	Applications of DSO..

Objectives: The student will learn that

1. Block and internal circuit diagram of CRO.
2. Use of CRO in laboratory.
3. Different types of CRO.

Outcomes: The student should be able to

1. Amplitude, Time, Phase difference and frequency measurement on CRO.
2. DSO and other CRO.
3. Component, probes testing by using CRO.

Model Questions

Q.1 Explain the function of delay Line in oscilloscope. What are the types of delay lines?

Q.2 What is intensity modulation? For what purpose it is used? Can phase and frequency be measured using intensity modulation ?

Q.3 Draw and explain the block diagram of digital storage oscilloscope.

Q.4 Explain analog storage oscilloscope. State the drawbacks of analog storage oscilloscope.

Q.5 Explain how an oscilloscope displays a signal.

Q.6 Compare Dual trace and Dual beam CRO.

Q.7 Draw the front panel of CRO and explain the functions of various controls.

Q.8 Explain the method of Lissajous patterns used for the frequency measurement.

Q.9 Draw and explain block diagram of CRO.

Q.10 Draw and explain block diagram of dual beam and dual trace CRO.

Q.11 Draw and explain circuit diagram of time base and triggered time base of CRO.

Chapterwise Plan

Subject Title: Electronic Instruments and Measurements

Chapter No. : 4

Chapter Name : Transducers for Displacement and Temperature Measurement

Approximate Time Needed : 8hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
27	Characteristics, requirement and selection of transducer.
28	Classification of transducer.
29	LVDT.
30	Stain Gauges.
31	Capacitive sensors.
32	RTD.
33	Thermistors.
34	Thermocouples.

Objectives: The student will learn that

1. Different electrical transduction principles employed in the measurement of various physical and mechanical parameters.
2. The practical aspects on the performance characteristics of thermocouples and digital reduction techniques.
3. The measurement of displacement and temperature by using different transducers.

Outcomes: The student should be able to

1. List requirement, selection criteria and classification of transducers.
2. Discuss resistive, inductive capacitive transducers are used for displacement measurement.
3. Discuss resistive transducer is used for strain measurement.
4. Discuss low and high temperature measurement transducers.

Model Questions

Q.1 Define gauge factor in stain gauge.

Q.2 What are the basic requirements of a transducer.

Q.3 Explain the working of strain gauge in detail. Derive the expression for gauge factor for strain gauge.

Q.4 What is LVDT? Explain and draw the complete constuctional diagram for it. state the specificatons, features and limitations of this transducer.

Q.5 Explain capacitive trasducer for displacement measurement.

Q.6 Explain thermocouple for temperature measuring in view of (i) Material used and Range (ii) Reference junction compensation (iii) Advantages and disadvantages over other types of temperature transducers.

Q.7 What is LVDT? Explain and draw the complete constructional diagram for it. State features and limitations of this transducer. Also explain the use of Phase Sensitive Detector (PSD) for operation of LVDT.

Q.8 Explain classification and selection criteria of transducer.

Q.9 Explain any one method of temperature measurement. Draw the diagram and explain advantages and disadvantages of the selected transducer. Write its output voltage equation.

Q.10 Explain stain gauge transducer. Derive its gauge factor. What are bounded and unbounded strain gauges and also explain advantages and disadvantages of semiconductor strain guage.

Q.11 Explain thermocouple with respect to following:-

(i) Temperature range

(ii) Material used

(iii) Linearity, non-linearity

(iv) Cold juction compensation

(v) Thermopiles.

Q.12 Differentiate sensors and transducers. Give two suitable examples of each.

Q.13 Explain the principle of RTD. Draw the 3-wire scheme for temparature measurement using RTD.

Q.14 Compare RTD, Thermocouple and Thermistor.

Chapterwise Plan

Subject Title: Electronic Instruments and Measurements

Chapter No. : 5

Chapter Name : Transducers for Pressure, Level and Flow Measurement

Approximate Time Needed : 10 hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
35	Dead weight tester.
36	McLeod Gauge, Pirani gauge.
37	Bourden tube, Diaphrame and Bellows.
38	Pressure Gauges.
39	Side glass tube and float type level methods.
40	Capacitance and ultrasonic type level methods.
41	Orifice plate and Venturi meter.
42	Rotameter.
43	Magnetic flow meter.
44	Turbine flow meter.

Objectives: The student will learn that

1. The analysis of force-balance devices, analysis of elastics diaphragms for both small and large deflections, practical aspects on the performance characteristics of level and flow techniques.
2. Pressure measuring devices- Principles, classifications, construction, working, specifications and applications.
3. Flow measuring meters- Principles, classifications, construction, working, specifications and applications.
4. Level transducers- Principles, classifications, construction, working, specifications and applications.

Outcomes: The student should be able to

1. Discuss pressure measurement methods.
1. Discuss classification of flow meters.
1. Discuss various level measurement methods.
2. Discuss sensors are used in different physical parameters
3. Discuss classification and selection of transducers.

Model Questions

Q.1 Explain the construction, working principle and operation of ultrasonic level measurement.

Q.2 Explain capacitive transducer for level measurement.

Q.3 Explain the construction, working principle and operation of Electromagnetic type of flow meter.

Q.4 Discuss any one of the techniques of measurement of high and low pressure measurement each.

Q.5 Explain side glass tube and float type level methods.

Q.6 Explain with neat diagram Dead weight tester, McLeod Gauge and Pirani gauge for pressure measurement.

Q.7 Explain elastic pressure transducer with diagram.

Q.8 Classify flow transducers and explain any one of them with neat diagram.

Q.9 Write a short note on- (i) Rotameter

(ii) Turbine flow meter.

Chapterwise Plan

Subject Title: Electronic Instruments and Measurements

Chapter No. : 6

Chapter Name : Data Acquisition and advances in Instrumentation Systems

Approximate Time Needed : 08hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
45	Indicators, Alarm.
46	Recorders.
47	Data logger.
48	DAS.
49	SCADA.
50	DCS.
51	PLC.
52	PLC applications.

Objectives: The student will learn that,

1. The analog data is generally acquired and converted to digital form for the purpose of processing, transmission, display and storage.
2. The supervisory control in which information sends and gathers to remote locations.
3. The distributed control system is a hardware and software package that encompasses all the functionality required to implement control and data acquisition functions.
4. The use of programmable logic controller in instrumentation system and in process industry.

Outcomes: The student should be able to

1) Describe the instrumentation aspects on data acquisition and processing.

2) Discuss the basic concept of PC based instrumentation system and its components.

3) Describe the advance instrumentation system like SCADA and DCS.

4) Discuss the various components are used in advance instrumentation system.

5) Discuss PLC and applications.

Model Question

Q.1 Draw a neat block diagram of multichannel analog multiplexed data acquisition system and explain its operation.

Q.2 Draw a neat block diagram of data logging system and hence differentiate with DAS? Give the advantages of data logging system.

Q.4 Explain with neat block diagram of multichannel data acquisition system to monitor temperature, flow, pressure, level, displacement and force.

Q.5 What are the objectives of Data Acquisition System?

Q.6 Write a short note on Data logger.

Q.7 Explain any one PC-based instrumentation system.

Q.8 Explain block diagram of PLC.

Q.9 Draw ladder diagram of bottle filling plant.

Q.10 Explain any one recorder.

Assignment – 1

1. Explain generalized instrumentation system in short.
2. How will you find the value of capacitance with the help of Schering Bridge? Explain with the help of derivation and vector diagram.
3. Explain the method of Lissajous patterns used for the frequency measurement.

Assignment – 2

1. Explain thermocouple for temperature measuring in view of (i) Material used and Range (ii) Reference junction compensation (iii) Advantages and disadvantages over other types of temperature transducers.
2. Explain the construction, working principle and operation of ultrasonic level measurement.
3. Explain with neat block diagram of multichannel data acquisition system to monitor temperature, flow, pressure, level, displacement and force.

Question Papers

S.E. sem3 (CBGS) (ETRX) 5/6/15

EIM

Q.P. Code : 4823

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question **No.1** is **Compulsory.**
(2) Attempt any **Three** questions from remaining **five** questions.

1. Solve All : **20**
   (a) Compare Maxwell bridge and Hey bridge for measurement of indutance.
   (b) Write the applications of instrument systems.
   (c) Write the specifications of CRO.
   (d) Explain level measurement by float type method.

2. (a) Discuss in detail static and dynamic characteristic of instruments. **10**
   (b) Write short note on "Data logger". **10**

3. (a) Explain the Kelvin double bridge for measurment of unknown resistance. **10**
   (b) Draw and explain the block diagram of DSO. **10**

4. (a) Explain in detail classification and selection criteria of transducer. **10**
   (b) Write short note on " Dead Weight Tester". **10**

5. (a) Draw and explain the block diagram of digital multimeter. **10**
   (b) Draw and explain the construction and working of magnetic flow meter. **10**

6. Write short notes on :- **20**
   (a) Monitoring instruments
   (b) Resistance temperature detector
   (c) Electronics volmeter using transistors
   (d) Capacitance sensor.

S.E Sem 3 (CBGS) ETRX 12/12/14.

EIM

QP Code :14672

[3 Hours] **[Total Marks:80**

N.B. (1) Question **No. 1** is compulsory.
(2) Attempt any **three** questions from remaining **five** questions.

1. Solve All:—
   (a) Draw and explain the working of practical Q- meter circuit. **20**
   (b) Define the following terms:—
      (i) Accuracy (ii) Resolution
      (iii) Hysteresis (iv) Calibration (v) Sensitivity
   (c) Estimate the bandwidth of CRO if a signal of 12 millisecond rise time is observed as the signal with 15 millisecond rise time.
   (d) Draw and explain the McLeod gauge for pressure measurement.

2. (a) Write short note on "Programmable logic controller". **10**
   (b) Draw and explain the turbine flow meter. **10**

3. (a) Explain the following terms related to thermocouples:— **10**
      (i) Law of intermediate metals.
      (ii) Law of intermediate temperature.
   (b) Draw and explain the block diagram of DSO. **10**

4. (a) Draw and explain the following bridges:— **10**
   (i) Maxwell bridge (ii) Schering bridge.
   (b) What are the types of errors in measurement system? Explain all in detail. **10**

5. (a) How the Lissajous figure are used for measurement of frequency and phase of the signal using CRO? Explain in detail. **10**
   (b) Draw and explain the construction and working of electronic voltmeter using transistors. **10**

6. Write short notes on:— **20**
   (i) Ultrasonic type level tranducers.
   (ii) Displacement measurement using potentiometers.
   (iii) Data acquisition system.
   (iv) Specification of CRO.

B.E- Electronics sem III (CBGS) M-2014
Sub:- EEM

QP Code : **NP-18752**
9/6/2014.

(3 Hours) **[Total Marks : 80**

N. B. : (1) Question **No. 1** is **compulsory**.
(2) Attempt any **three** questions from remaining **five** questions.

1. Solve **all** :– **20**
   (a) Explain the remedies to eliminate the errors in measurement.
   (b) Write the specifications of analog multimeter.
   (c) Discuss the role of delay line in CRO.
   (d) Draw and explain the venturi meter for flow measurement.

2. (a) Write short note on "Data Acquisition System". **10**
   (b) Draw and explain the construction and working of dead weight tester. **10**

3. (a) Explain in detail voltage and frequency measurement using CRO. **10**
   (b) Write short note on "Q-meter". **10**

4. (a) Explain the static and dynamic characteristics of instruments in detail. **10**
   (b) Draw and explain the block diagram of DSO. Also write its applications. **10**

5. (a) Draw and explain the construction and working of linear variable differntial transformer. **10**
   (b) Draw and explain the construction and working of Rotameter. **10**

6. Write short note on :– **20**
   (i) Mega ohm bridge
   (ii) Chop and Alternate mode of CRO
   (iii) Capacitance sensor for displacement measurement
   (iv) Float type method of level measurement.

(ETRX) 16/12/13
S. E. SEM - III (CBGS)
Electronic Instruments and Measurement

28-11-2013-DTP-P-7-RA-6

Con. 9938-13. **GX - 12182**

(3 Hours) **[Total Marks :80**

N. B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **three** questions from remaining **five** questions.
(3) Assume **suitable** data if **required**.

1. Solve all :— **20**
   (a) Define the following terms :–
   (i) Accuracy (ii) Precision (iii) Sensitivity
   (iv) Linearity (v) Resolution.
   (b) Write the applications of Q-meters.
   (c) Estimate the Bandwidth of CRO if a signal of 12 MS rise time is observed as the signal with 15 MS rise time.
   (d) Write the selection criteria of transducer.

2. (a) Write short notes on "Resistance strain guages". **10**
   (b) List the types of liquid level measurements. Explain any two in detail. **10**

3. (a) Compare the temperature measurement transducers RTD, thermistors and thermcouples on the basis of principle, characteristics, ranges and applications. **10**
   (b) Explain the magnetic flow meter in detail. **10**

4. (a) Draw and explain the block diagram of data logger. **10**
   (b) Explain the measurement of unknown rcsistance using Kelvin double bridge. **10**

5. (a) Draw and explain the generalized block diagram of the CRO. **10**
   (b) Explain the linear variable differential transformer in detail. **10**

6. (a) What are the types of errors in measurements ? Explain all in details. **10**
   (b) Draw and explain the block diagram of digital storage oscilloscope. Also write the applications of DSO. **10**

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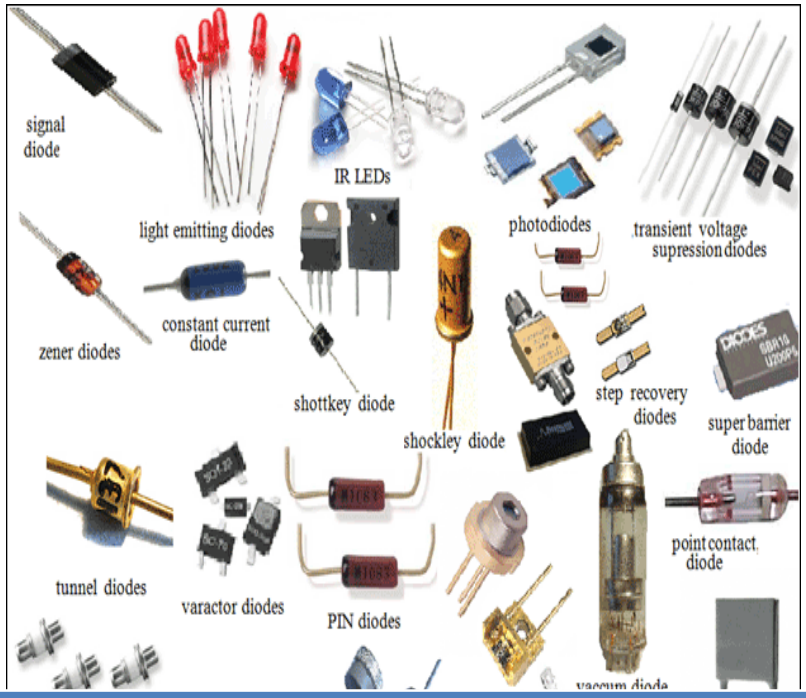

ELECTRONIC DEVICES

PADMABHUSHAN VASANTDADA PATIL PRATISHTHAN'S
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1990
COLLEGE OF ENGINEERING
Empower The Knowledge

Mrs. RADHA WANODE

Subject Plan

GROUP NAME : ELECTRONIC DEVICES AND CIRCUITS
COURSE TITLE: Electronic Devices **COURSE CODE : EXC 302** **SEM : III (SH 2015)**
PRE-REQUISITE : Basic Electrical & Electronics Engineering

RATIONALE

Electronic devices enable the students to comprehend the theory, concepts, characteristics and working principles of basic electronic devices and their applications in electronic circuits. The knowledge of various devices acquired by the students will help them to design, test, troubleshoot and repair electronic circuits.

OBJECTIVES :

1. To deliver the knowledge about physics of basic semiconductor devices
2. To enhance comprehension capabilities of students through understanding of electronic devices
3. To introduce and motivate students to the use of advanced microelectronic devices
4. To create foundation for forthcoming circuit design
5. To prepare the students to excel in post graduate studies.

OUTCOME :

The student should be able to

1. Ability to analyze characteristics of semiconductor junctions
2. Ability to differentiate between bipolar and unipolar conduction
3. Ability to understand physics of optical devices
4. Ability to understand working principle of power devices

LEARNING RESOURCES: -

RECOMMENDED BOOKS: -

1. Donald A. Neamen, "*Semiconductor Physics and Devices*" Tata MCGraw Hill, Third Edition
2. S. M. Sze, "*Semiconductor Devices: Physics and Technology*", Wiley, Second Edition
3. Sung-Mo Kang,YusufLeblebici, "*CMOS Digital Integrated Circuits*", Tata McGraw Hill, Third Edition
4. David Bell, "Electronic Devices and Circuits", Oxford, Fifth Edition.
5. S Slivahanan and N. Suresh Kumar, "*Electronic Devices and Circuits*", McGraw Hill, Third Edition
6. Gordon W. Roberts and Adel S. Sedra, "*Spice*", Oxford, Second Edition

COURSE MATERIALS MADE AVAILABLE

1. Course instructional objectives & outcomes
2. Syllabus
3. Chapter wise Question Bank

Evaluation :

Theory Exam	80 M
Internal assessment:-. The average marks of Mid-term test (20 M) & End-term test (20 M) will be considered as final IA marks	20 M
Practical and Oral	50 M
Term Work	25 M
Total	175 M

List of Experiments:

Atleast 10 experiments based on the entire syllabus

Expt. No.	Name of the Experiments
1	Forward & reverse Bias of PN junction diode
2	Zener diode as voltage regulator
3	Frequency response of CE BJT amplifier
4	Frequency response of CS FET amplifier
5	Characteristics of photodiode
6	Characteristics of SCR
7	Simulate characteristics of BJT, JFET, MOSFET by using LT-Spice
8	Simulate frequency response of BJT by using LT-Spice
9	Simulate frequency response of FET by using LT-Spice
10	Simulate frequency response of MOSFET by using LT-Spice
11	Simulate characteristics DIAC, TRIAC by using LT-Spice

Chapterwise Plan

Subject Title: Electronic Devices

Chapter No. : 1

Chapter Name : Junction Analysis

Approximate Time Needed : 14 hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
1	Introduction
2	PN junction Diode: Basic Structure, Energy Band Diagrams, Zero Applied Bias
3	Applied Bias, Reverse Applied Bias,
4	PN Junction current, Small signal model of PN junction
5	Generation and recombination of currents, junction breakdown.
6	Zener Diode: Breakdown mechanisms, Characteristics, Effect of Temperature
7	Application as voltage regulator and backward diode
8	Varactor diode: Working and characteristics
9	Tunnel diode: V-I Characteristics and working
10	TED (Transferred Electron Device): Basic concept, Negative differential resistance
11	IMPATT: Static and Dynamic Characteristics
12	Schottkey barrier diode: Qualitative characteristics, Ideal junction properties, Nonideal effects on barrier height and V-I characteristics
13	Ideal Non rectifying barriers, Tunneling Barrier, Specific contact resistance
14	Heterojunction materials, Energy Band Diagrams, Two dimensional electron gas.

Objectives:

The student will learn that,

1) Different types of biasing used in semiconductor diodes, their characteristics, and energy band diagram
2) Different types of diodes in detail.
3) Barrier produces at junction of various diodes.
4) Ohmic and non-Ohmic junctions in diodes

Lesson Outcome:

The student should be able to,

1) Identify types of diode according to characteristics.
2) Design circuit according to need using diodes.

Model Questions:

1. Justify that the space charge width increase with reverse biased voltage in a pn junction diode

2. Derive equation of built in potential Vbi for a p– n junction under Zero bias and hence calculate Vbi at T = 300 k for Nd = 10^{15} cm^{-3} and Na = 10^{15} cm^{-3}.

3. An abrupt PN junciton has dopant concentrations of
Na = 2×10^{16} cm^{-3} and Nd = 2×10^{15} cm^{-3} at T = 300 K
Calculate: (a) Vbi
(b) W at $V_R = 0$ and $V_R = 8V$
(c) E maximum at $V_R = 0$ and $V_R = 8V$

4 Derive equation of built in potential Vbi for a p– n junction under Zero bias and hence calculate Vbi at T = 300 k for Nd = 10^{15} cm^{-3} and Na = 10^{15} cm^{-3}.

5. Explain construction and VI characteristics of Tunnel diode.

6. Explain schottky barrier diode with the help of energy band diagram

7. Explain concepts, construction, characteristics and working of Gunn diode.

8. Explain the need of Hetero junction, explain the terms straddling, staggered and broken gap in relation to hetero junction. Explain the quantization of energy of an electron in a potential well in hetero junction. Explain this concept with respect to the ideal energy band diagram of an nN Ga As–Al Ga As hetero junction in thermal equilibrium.

Chapterwise Plan

Subject Title: Electronic Devices

Chapter No. : 2

Chapter Name : Bipolar Devices

Approximate Time Needed : 08 hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
15	The bipolar transistor action
16	Minority carrier distribution
17	Low-frequency common-base current gain
18	Non-ideal effects, Ebers-Moll Model
19	Gummel-Poon Model
20	Hybrid-Pi Model, Frequency Limitations
21	Current gain in HBT
22	Basic n-p-n HBT structure with band diagram

Objectives:

The student will learn that

1. Bipolar Junction Transistor in detail.
2. Low frequency analysis model for BJT.
3. Hybrid pi model of BJT.
4. Current gain in HBT and n-p-n HBT Structure.

Lesson Outcomes:

The student should be able to understand

1) Characteristic of BJT.
2) Different types BJT structure, different configuration used in BJT.

3) Current gain and voltage gain basics.
4) Dc analysis and AC analysis basics and need.
5) Basic n-p-n HBT structure with band diagram

Model Questions:

1. What is primary advantage of HBT over BJT? Draw and explain schematic cross section of an npn HBT structure with its energy band diagram when HBT is operated under active mode?

2. What is Non ideal effects in BJT and hence explain Base width modulation in brief

3. Explain basic principle of operation of BJT with the help of construction, minority carrier distribution and energy band diagram.

4. What is HBT, Explain construction and energy band diagram of HBT?

5. Describe the time delay factors in the frequency limitation of the bipolar transistor, calculate the emitter–collector transit time, cut off frequency and the beta cut off frequency of a bipolar transistor, with the following parameters, consider a silicon npn transistor at T = 300 K with a low frequency common emitter current gain of $\beta = 100$. Assume the following parameters :–

$IE = 50\mu A, C_{je} = 0.40\,PF, C\mu = 0.05\,PF$

$X_B = 0.5\,\mu m, D_n = 25\,cm^2/s, X_{dc} = 2.4\,\mu m$

$r_C = 20\Omega, C_S = 0.1\,\rho F$

Chapterwise Plan

Subject Title: Electronic Devices

Chapter No. : 3

Chapter Name : Field Effect Devices

Approximate Time Needed : 16 hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
23	JFET: Construction
24	operation and device characteristics
25	V-I relationship and transconductance
26	Small signal equivalent model
27	frequency limitation factors and cut-off frequency
28	MOSFET: Two terminal MOS structure
29	MOSFET construction,
30	Band diagrams under equilibrium and external bias,
31	Threshold Voltage, V-I and CV characteristics
32	Channel length modulation, Short Channel effects,
33	MOSFET Model
34	MESFET: Device structure, principle of operation,
35	V-I characteristics
36	High frequency performance
37	MODFET (i.e. HEMT) : Fundamentals,
38	V-I Characteristics, Cut-off Frequency

Objectives:

The student will learn that

4. Construction, operation and device characteristics of JFET
5. Small signal equivalent model of JFET.
6. Two terminal MOS structure and MOSFET construction.
7. MESFET & MODFET structure and operation.

Lesson Outcomes:

The student should be able to

1) Identify field effect devices based on their characteristics.
2) Tanscoductance parameter with respect to JFET.
3) Design circuit with the help of FET.

Model Questions:

1. Explain difference between N-channel and P-channel JFET, also explain characteristics for N-channel JFET.

2. Explain in brief TWO terminal MOS structure.

3.

For an n-channel MOS transistor with
$\mu n = 600\ cm^2/V.S$, $Cox = 7 \times 10^{-8}\ F/cm^2$,
$W = 20\mu m$, $L = 2\mu m$ and $VTO = 1.0\ V$.
Examine the relationship between the Drain current and terminal voltages.

4. Explain structure and operation of MOSFET considering different cases of threshold voltage VT.

5.

Discuss the device structure and principle of operation of MESFET. Derive the equation for current–voltage characteristics for MESFET. Describe the various regions of operation on V–I characteristics.

6.

Draw band diagrams for accumulation, depletion and inversion regions for MOS capacitor.

Calculate threshold voltage for a polysilicon gate n–channel MOS transistor with substrate at Zero potential with the following parameters :–

Substrate doping density $N_A = 10^{16}\ cm^{-3}$

Polysilicon gate doping density $N_D = 2 \times 10^{20}\ cm^{-3}$

Gate oxide thickness $t_{ox} = 500^0\ A$

Oxide–interface fixed charge density $N_{OX} = 4 \times 10^{10}\ cm^{-2}$

7. Explain construction, working and characteristics of N channel JFET, explain frequency limitation factor.

Chapterwise Plan

Subject Title: Electronic Devices

Chapter No. : 4

Chapter Name : Optical Devices

Approximate Time Needed : 06 hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
39	Photon absorption coefficient.
40	EHP generation rate
41	PN junction solar cell, amorphous silicon solar cell
42	Photodetectors**,** Photoconductor, photodiode
43	PIN photodiode, phototransistor
44	Optocouplers, Operation, construction, specifications and applications

Objectives:

The student will learn that

1) Solar cell, their need and operation.
2) Photo detector, their need and operation.
3) Optocouplers, their need and operation.

Lesson Outcomes

The student should be able to

1) Understand photon absorption parameter.
2) Understand need of isolation in electrical devices.
3) To design circuit with photodiode.
4) Identify different types of solar cells.

Model Questions:

1. Justify how phototransistor is more practical than photo diode.

2.

What is photovoltaic effect. Explain in detail Solar Cell with working, characteristics and practical applications.

3. sketch the VI characteristics of a PN junction solar cell

4. Describe construction, working and characteristics of photodiode and Avalanche Photodiode.

Chapterwise Plan

Subject Title: Electronic Devices

Chapter No. : 5

Chapter Name : Power Devices

Approximate Time Needed : 08 hrs

Lesson Schedule :

Lecture No.	Portion covered per hour
45	PNPN Diode: Basic structure and characteristics
46	SCR: Basic structure, characteristics, Two transistor analogy.
47	DIAC and TRIAC: Basic Structure and characteristics
48	GTO: Basic structure and characteristics
49	PUT: Operation and characteristics
50	UJT: Operation, characteristics, parameters
51	UJT as a relaxation oscillator
52	IGBT: Device structure, equivalent circuit and characteristics

Objectives:

The student will learn that

1) Basic structure of power devices.
2) Need of power devices in engineering
3) Types of power devices.
4) Operation and characteristics of different power devices.

Outcomes:

The student should be able to

1) Compare different types of power devices.
2) Draw and explain characteristics of different power devices.

Model Questions:

1. Explain construction and characteristics of UJT.
2. Explain construction, characteristics and working of TRIAC and DIAC.
3. explain concepts, construction, characteristics and working of SCR
4. Describe construction and VI characteristics of IGBT.

Assignments

ASSIGNMENT 1 (DATE: 6th Aug 2015)

1. Explain construction and VI characteristics of Tunnel diode.
2. Explain concepts, construction, characteristics and working of Gunn diode
3. Explain basic principle of operation of BJT with the help of construction, minority carrier distribution and energy band diagram.
4. What is HBT, Explain construction and energy band diagram of HBT?

ASSIGNMENT 2 (DATE: 14th Sep 2015)

1. What is photovoltaic effect? Explain in detail Solar cell with working and characteristics.

2. Describe construction, working and characteristics of photodiode and Avalanche Photodiode. Explain construction and characteristics of UJT.

3. Explain construction, characteristics and working of TRIAC and DIAC.

4. explain concepts, construction, characteristics and working of SCR

5. describe construction and VI characteristics of GTO

S.E. ETRX sem III CBGS Nov-13 25/11/13

Sub:- Electronics Devices.

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Con. 7851-13. **GX-12037**

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. **1** is **compulsory** and solve any **three** questions from **remaining** questions.
(2) Assume **suitable** data if **necessary**.
(3) Draw **neat** and clean **figures**.

Given Data :–

$q = 1.6 \times 10^{-19}$ C
$k = 1.38 \times 10^{-23}$ J/K
$\eta i = 1.5 \times 10^{10}$ cm^{-3}
$\epsilon_{si} = 11.7 \times 8.854 \times 10^{-14}$

1. (a) Justify that the space charge width increase with reverse biased voltage in a p–n junction diode. **5**

   (b) Sketch low frequency capacitance versus gate voltage of a MOS capacitor with n–type substrate show individual capacitance components. **5**

   (c) Sketch the IV characteristics of a PN junction solar cell. **5**

   (d) Describe construction and V–I characteristics of IGBT. **5**

2. (a) Derive equation of built in potential Vbi for a p– n junction under Zero bias and hence calculate Vbi at T = 300 k for Nd = 10^{15} cm^{-3} and Na = 10^{15} cm^{-3}. **10**

   (b) What is primary advantage of HBT over BJT? Draw and explain schematic cross section of an npn HBT structure with its energy band diagram when HBT is operated under active mode? **10**

3. (a) Explain construction and V–I characteristics of Tunnel diode. **10**

   (b) Explain construction, working and characteristic of N–channel JFET, explain frequency limitation factors. **10**

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Con. 7851-GX-12037-13. 2

4. (a) Draw band diagrams for accumulation, depletion and inversion regions for MOS capacitor. **10**
Calculate threshold voltage for a polysilicon gate n–channel MOS transistor with substrate at Zero potential with the following parameters :–
Substrate doping density $N_A = 10^{16}\ cm^{-3}$
Polysilicon gate doping density $N_D = 2 \times 10^{20}\ cm^{-3}$
Gate oxide thickness $t_{ox} = 500^0\ A$
Oxide–interface fixed charge density $N_{ox} = 4 \times 10^{10}\ cm^{-2}$

(b) Describe the time delay factors in the frequency limitation of the bipolar transistor, calculate the emitter–collector transit time, cut off frequency and the beta cut off frequency of a bipolar transistor, with the following parameters, consider a silicon npn transistor at T = 300 K with a low frequency common emitter current gain of $\beta = 100$. Assume the following parameters :– **10**

$IE = 50\mu A, C_{je} = 0.40\ PF, C\mu = 0.05\ PF$

$X_B = 0.5\ \mu m, D_n = 25\ cm^2/s, X_{dc} = 2.4\ \mu m$

$r_C = 20\Omega, C_S = 0.1\ \rho F$

5. (a) Describe construction, working and characteristic of :– **10**
(i) Photodiode and
(ii) Avalanche Photodiode.

(b) Discuss the device structure and principle of operation of MESFET. Derive the equation for current–voltage characteristics for MESFET. Describe the various regions of operation on V–I characteristics. **10**

6. (a) Explain construction, working and characteristics of SCR. **10**

(b) Explain the need of Hetero junction, explain the terms straddling, staggered and broken gap in relation to hetero junction. Explain the quantization of energy of an electron in a potential well in hetero junction. Explain this concept with respect to the ideal energy band diagram of an nN Ga As–Al Ga As hetero junction in thermal equilibrium. **10**

31/05/2014 Electronic Devices (CBGS) SE ETRX II 31 May 2014

QP Code : **NP-18616**

(3 Hours) [Total Marks : 80

N. B. : (1) Question No. **1** is compulsory and solve **any three** questions from remaining questions.
(2) Assume suitable data if necessary.
(3) Draw neat and clean figures.

Given Data —
(1) $q = 1.6 \times 10^{-19}$ C
(2) $k = 1.38 \times 10^{-23}$ J/K
(3) $ni = 1.5 \times 10^{10}$ cm^{-3}
(4) $\epsilon si = 11.7 \times 8.854 \times 10^{-14}$

1. (a) What is Non-ideal effects in BJT and hence explain Base width modulation in brief. **5**

1. (b) Justify how phototransistor is more practical than photo diode. **5**

1. (c) Explain in brief TWO Terminal MOS structure. **5**

1. (d) Explain construction and characteristics of UJT. **5**

2. (a) Explain concepts, construction, characteristics and working of Gunn diode. **10**

2. (b) Explain basic principle of operation of BJT with the help of construction, minority carrier distribution and energy band diagrams. **10**

3. (a) Explain structure and operation of MOSFET considering different cases of threshold voltage V_T. **10**

(b) An abrupt PN junciton has dopant concentrations of **10**
$Na = 2 \times 10^{16}$ cm^{-3} and $Nd = 2 \times 10^{15}$ cm^{-3} at T = 300 K
Calculate: (a) Vbi
(b) W at $V_R = 0$ and $V_R = 8V$
(c) E maximum at $V_R = 0$ and $V_R = 8V$

4. (a) What is photovoltaic effect. Explain in detail Solar Cell with working, characteristics and practical applications. **10**

4. (b) For an n-channel MOS transistor with **10**
$\mu n = 600\ cm^2/V.S$, $Cox = 7 \times 10^{-8}\ F/cm^2$,
$W = 20\mu m$, $L = 2\mu m$ and $VTO = 1.0\ V$.
Examine the relationship between the Drain current and terminal voltages.

5. (a) Explain construction, working and characteristics of TRIAC & DIAC. **10**

5. (b) Explain schottky-barrier diode with the help of energy band diagram. **10**

6. (a) What is HBT, Explain construction and energy band diagram of HBT. **10**

6. (a) Explain difference between N-channel and P-channel JFET, Also explain characteristcs (Drain and Transfer) for N-channel JFET. **10**

S.E. ETRX Sem(3) (CBGS) · 20/11/14.
Electronic Devices.

QP Code : 14541

(3 Hours) **[Total Marks : 80**

N.B. : (1) question **No. 1** is **compulsary** and solve any **Three** questions from remaining questions.
(2) Assume suitable data if necessary.
(3) Draw neat and clean figures.

1. Answer any **five** :
   (a) For the diodes, define forward voltage drop, maximum forward current, dynamic resistance, reverse saturation current & reverse brakdown voltage. **5**
   (b) Draw characteistics of Pn junction in thermal equilibrium? Explain. **5**
   (c) Define the contributing factors forwards the low frequency common base current gain of BJT. **5**
   (d) Define internal pinchoff voltage, pinchoff voltage & drain to source saturation voltage fo JFET, **5**
   (e) What are types of MOSFET? Explain. **5**
   (f) Explain consturction working & characeristics of UJT. **5**

2. (a) What is space charge width? Derive an expression for it, when the diode is forward biased and reverse biased. **10**
   (b) List the ideal conditions of BJT and explain the non-ideal effects. **10**

3. (a) Draw Ebers - Moll equivalent circuit of BJT & derive mecessary expressions for current and voltages. **10**
   (b) Compare BJT, JFET & MESFET. **10**

(4) (a) What is channel length modulation in MOSFET? Derive necessary expression for the same. **10**
   (b) Explain construction, working & characeristics ot Tunnle diode - **10**

5. (a) What is HBT ? Explain construction & energy band diagram of the same. **10**

   (b) for an n - channel MOS transistor with $\mu n = 600\ cm^2/vs$, $C_{ox} = 7 \times 10^{-8}\ F/cm^2$, W = 20 μm, L = 2 μm and $V_{TO} = 1.0V$ Examine the relationship between the drain current & terminal voltages. **10**

6. Write short notes **20**
   (a) SCR
   (b) Solar Cell
   (c) Photo diode
   (d) IGBT

S.E. Sem III (CBGS) ETRX
Electronic Devices 14/5/.

Q.P. Code : 4812

(3 Hours) **[Total Marks : 80**

N.B. : (1) Questions No.1 is **compulsory** and Solve any **three** questions from the remaining questions.
(2) Assume suitable data if necessary.
(3) Draw neat and clean Figures.

1. (a) What are nonideal effects in BJT? Explain any one nonideal eftect in BJT. **5**
   (b) Determine the ideal reverse saturation current density in silicon P-N diode at 300^{0}k Given Na=Nd=10^{16} cm^{-3}, ni=1.5x$10^{10}$$cm^{-3}$ Dn=25cm^2/s εr=11.7, Dp=10cm^2/s τpo= τno=5x10^{-7}s **5**
   (c) With neat diagram explain the operation of UJT relaxation oscillator. **5**
   (d) Compare photodiode with phototransistor. **5**

2. (a) Draw energy band diagram of P-N junction for zero, forward, reverse bias clearly showing junction diagram, depletion width, fermi energy level and barrier potential. **10**
   (b) Calculate the theoretical barrier height, built in potential barrier and maximum electric field in a metal semiconductor diode for zero applied bias- Consider a contact between tungsten and n type silicon doped to Nd=$10^{16}$$cm^{-3}$ at T=300k. The metal work function for tungsten is ϕm=4.55V and electron a affinity for silicon is x=4.01V. Nc=2.8x$10^{19}$$cm^{-3}$, K=1.38x$10^{-23}$J/K, εs=11.7x8.85x$10^{-14}$, e=1.6x$10^{-19}$c **10**

3. (a) Calculate the threshold voltage V_{TO} at V_{SB}=0, for a polysilicon gate n channel MOS transistor with the following parameters - substrate doping density NA=$10^{16}$$cm^{-3}$ polysilicon gate doping density ND=2x$10^{20}$$cm^{-3}$ gate oxide thickness tox=500A^0 oxide Interface fixed charge density NOX=4x$10^{10}$$cm^{-2}$ **10**
   (b) Derive the drain current equation ID for MOSFET in ohmic and saturation regions. **10**

4. (a) Draw and explain construction, working, characteristics of JFET. Explain frequency limitation factors. **10**
   (b) Explain, schottky effect. Derive the position of maximum barrier Xm. **10**

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Q.P. Code : 4812

2

5. (a) Draw and explain, construction and working of : 10
   (i) HEMT (MODFET)
   (ii) MESFET
   (b) Explain basic structure and characteristics of : 10
   (i) SCR (ii) DIAC

6. Solve any **four** of the following : 20
   (a) Draw and explain Ebers-moll model of transistor.
   (b) With the help of circuit diagram and characteristics explain application of zener diode as a voltage regulator.
   (c) What are optocouplers? Explain any one application of optocoupler.
   (d) Sketch and explain V-I and C-Vcharacteristics of MOSFET
   (e) Explain channel length modulation with cross section of MOSFET. Write equation associated with this effect.

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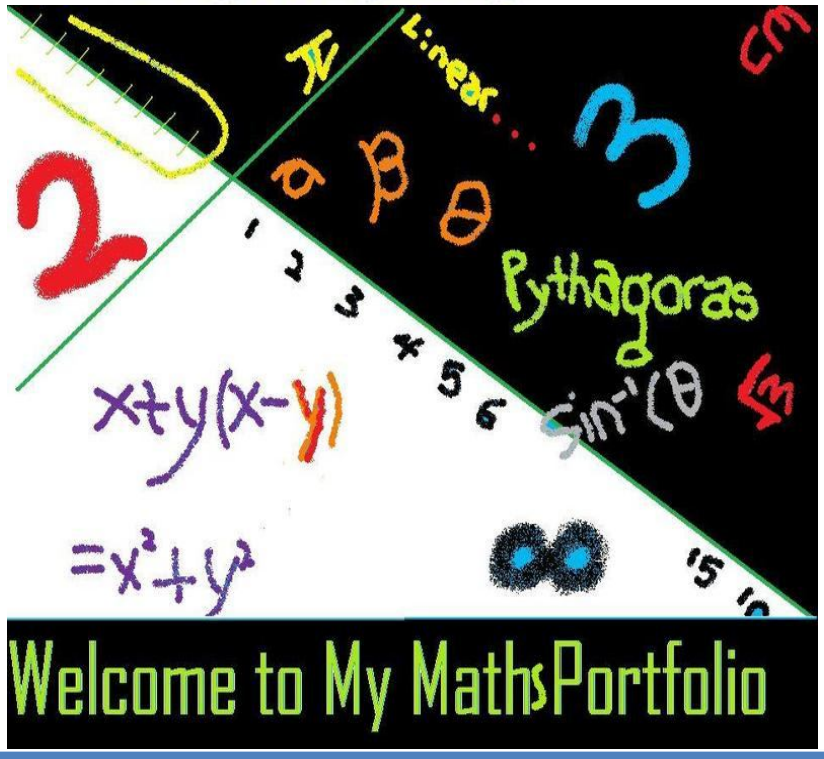

SH 2015

MATHEMATICS III

PADMABHUSHAN VASANTDADA PATIL PRATISHTHAN'S
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Empower The Knowledge

Mrs. Arti Kurkure

Vector Integration

Line Integral

Problems

01)If $\vec{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$ evaluate the line integral $\int_C \vec{A} \circ d\vec{r}$ from (0,0,0) to (1,1,1) along the following paths C:-

a) $x = t, y = t^2, z = t^3$ b) the line segment joining (0,0,0) to (1,1,1)

c) the line segment joining (0,0,0) to (1,0,0), then to (1,1,0), then to (1,1,1)

02) Evaluate $\int_C \vec{A} \circ d\vec{r}$ along the curve C with

$\vec{r} = a\cos\theta\hat{i} + a\sin\theta\hat{j} + b\theta\hat{k}$ from $\theta = \frac{\pi}{4}$ to $\theta = \frac{\pi}{2}$ If

$\vec{A} = (-3a\sin^2\theta\cos\theta)\hat{i} + a(2\sin\theta - 3\sin^3\theta)\hat{j} + b\sin 2\theta\hat{k}$

03)Find the work done under force $\vec{F} = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$ in moving a particle once around

a) the circle $x^2 + y^2 = 9, z = 0$ b)the ellipse $9x^2 + 4y^2 = 36, z = 0$

04) Evaluate $\int_C \vec{A} \circ d\vec{r}$ along the curve $x^2 + y^2 = 1, z = 1$ in the positive direction from (0,1,1) to (1,0,1) If

$\vec{A} = (2x + yz)\hat{i} + zx\hat{j} + (xy + 2)\hat{k}$

05) Evaluate $\int_C \vec{A} \circ d\vec{r}$ along the curve C with position vector

$\vec{r} = a\cos\theta\hat{i} + b\sin\theta\hat{j} + c\theta\hat{k}$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$ If $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$

06) Find the work done under force $\vec{F} = 3xz\hat{i} - 4y\hat{j} + z\hat{k}$ in moving a particle along the curve $x = t^2 + 1, y = t^3, z = 2t + 3$ from (1, 0, 3) to (2, 1, 5)

07) Evaluate $\int_C \vec{F} \times d\vec{r}$ where $\vec{F} = xy\hat{i} - z\hat{j} + x^2\hat{k}$ where C is the curve $x = t^2, y = 2t, z = t^3$ from t=0 to t=1 **(D-10)**

Conservative Vector Field

Problems

08)Show that $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$ is conservative. Find its scalar potential ϕ.Hence find the work done in moving a particle in this field of force $\vec{F}$ from the point (0,1,-1) to the point $(\frac{\pi}{2}$,-1,2)

09) Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is irrotational and hence find its scalar potential ϕ. Hence find the work done in moving a particle in this field of force $\vec{F}$ from the point (1,-2,1) to the point (3,1,4) **(M-09)**

10) Find the constants a, b, c if $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational. Find its scalar potential ϕ such that $\vec{F} = \nabla\phi$.Hence find the work done in moving a particle in this field of force $\vec{F}$ from the point (1,2,-4) to the point (3,3,2) along the line joining these two points

11) Show that $\vec{F} = (ye^{xy} \cos z)\hat{i} + xe^{xy} \cos z\hat{j} - e^{xy} \sin z\hat{k}$ is conservative. Find its scalar potential ϕ .Hence find the work done in moving a particle in this field of force $\vec{F}$ from the point (0,0,0) to the point (-1,2, π) **(M-08,D-10)**

12)Show that $\vec{F} = xyz^2\hat{i} + (x^2z^2 + z\cos yz)\hat{j} + (2x^2yz + y\cos yz)\hat{k}$ is conservative. Find its scalar potential ϕ.Hence find the work done in moving a particle in this field of force $\vec{F}$ from the point (0,0,1) to the point $(1,\frac{\pi}{4}$,2).

13)Show that $\int_P^Q (2xy^3 - y^2 \cos x)dx + (1 - 2y \sin x + 3x^2y^2)dy = \frac{\pi^2}{4}$

along the arc $2x = \pi y^2$ from P(0,0) to Q$(\frac{\pi}{2},1)$

Green's Theorem

Problems

14) Verify Green's theorem in the plane for

a) $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region enclosed by the curves $y = x^2$ and $x = y^2$ **(D-08)**

b) $\oint_C (x^2 - 2xy)dx + (x^2y + 3)dy$ where C is the boundary of the region enclosed by the curves $y^2 = 8x$ and $x = 2$ **(M-08)**

c) $\oint_C (y - \sin x)dx + \cos x dy$ where C is the triangle with vertices at (0, 0), ($\frac{\pi}{2}$,0) and ($\frac{\pi}{2}$,1)

d) $\oint_C (x^2 - y^3)dx + (x^3 + y^2)dy$ where C is the circle $x^2 + y^2 = 64$

15) Using Green's theorem evaluate

a) $\int_C \vec{A} \circ d\vec{r}$ where $\vec{A} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$ and C is the circle with center at (3, 3) and radius 1

b) $\int_C \vec{A} \circ d\vec{r}$ where $\vec{A} = (3x + 4y)\hat{i} + (2x - 3y)\hat{j}$ and C is the circle $x^2 + y^2 = 2^2$ **(May-09)**

c) $\oint_C x^2 y dx + xy^2 dy$

d) $\oint_C (2x^2 - y)dx + (2x + y^2)dy$ over C where Cis the boundary of the region bounded by $y = x^2$, y=1 and x=0 **(D-10)**

e) the area A bounded by a simple closed curve C in the xy-plane and hence find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

16) Verify Green's theorem in the plane for

a) $\oint_C (x^2 - xy)dx + (x^2 - y^2)dy$ where C is the boundary of the region enclosed by the curves $y = x^2$ and $y = x$

b) $\oint_C x^2 dx - xydy$ where C is the triangle with vertices at (0, 2), (2, 0) and (4, 2)

c) $\oint_C (x^2 - \cosh y)dx + (y + \sin x)dy$ where C is the rectangle with vertices at (0, 0), (π ,0),(π, 1)and (0,1)

d) $\oint_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the region enclosed by

(i)the x-axis and the 3semi-circle $y = \sqrt{1 - x^2}$ (ii)the lines x=0,x=2,y=0,y=3**(M-09)**

17) Using Green's theorem evaluate

a) $\int_C \vec{A} \circ d\vec{r}$ where $\vec{A} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$ and C is some circle enclosing the origin

b) $\oint_C (e^{x^2} - xy)dx - (y^2 - x)dy$ where C is the circle $x^2 + y^2 = 1$ **(D-09)**

c) $\oint_C (2x - y^3)dx - xydy$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ **(M-09)**

d) find the area of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

f) find the area of the lemniscate $(x^2 + y^2)^2 = a^2(x^2 - y^2)$

Surface Integral

Problems

14)Evaluate $\iint_S \vec{A} \circ d\vec{S}$ where and S is that part of the plane

a) 2x+3y+6z=12 which is located in the first octant and $\vec{A} = 18z^2\hat{i} - 12\hat{j} + 3y\hat{k}$ **(D-08)**

b) 2x+y+z=6 which is in the first octant and $\vec{A} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ **(M-08)**

15) Evaluate $\iint_S \vec{A} \circ d\vec{S}$ where $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is the sphere $x^2 + y^2 + z^2 = 1$ which in located in the first octant.

16) Evaluate $\iint_S \vec{A} \circ d\vec{S}$ where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between the planes z=0 and z=5.

17) Evaluate $\iint_S \vec{A} \circ d\vec{S}$ where $\vec{A} = 2y\hat{i} - z\hat{j} + x^2\hat{k}$ and S is the surface of the parabolic cylinder located in the first octant bounded by the planes y=4 and z=6

18) Evaluate $\iint_S \vec{A} \circ d\vec{S}$ where $\vec{A} = 4xz\hat{i} + xyz\hat{j} + 3z\hat{k}$ and over the entire region above the xy-plane bounded by the cone $x^2 + y^2 = z^2$ and the plane z=4.

Gauss Divergence Theorem

Problems

19)Verify Gauss divergence theorem for

a) $\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ over the cube bounded by the planes x=0, x=1, y=0, y=1, z=0 and z=1

b) $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$

c) $\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ over the cylinder $x^2 + y^2 = 4, z = 0, z = 3$ **(M-08)**

20) Use the divergence theorem to evaluate

a) $\iint_S \left[(x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}\right] \circ d\vec{S}$ over the cube bounded by the planes x=0, x=1, y=0,y=1,z=0 and z=1

b) $\iint_S \nabla r^2 \circ d\vec{S}$ where S is the sphere $x^2 + y^2 + z^2 + 2x + 6y + 1 = 0$

c) $\iint_S [(xz^2\hat{i} + (x^2y - z^3)\hat{j} + (2xy + y^2z)\hat{k}] \circ d\vec{S}$ where S is the surface enclosing the region bounded by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane z=0

21) Verify Gauss divergence theorem for

a) $\vec{A} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the surface of the parallellopiped bounded by the planes x=0, x=a, y=0, y=b, z=0 and z=c.

b) $\vec{A} = 2x^2\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ over the region in the first octant bounded by the

cylinder $y^2 + z^2 = 9$ and $x = 2$ **(D-08)**

c) $\vec{A} = 4xz\hat{i} + xyz^2\hat{j} + 3z\hat{k}$ over the closed region above the xy-plane bounded by the cone $x^2 + y^2 = z^2$ and the plane z=4

d) $\vec{A} = 4x\hat{i} + 3y\hat{j} - 2z\hat{k}$ over the closed surface bounded by the planes 2x+2y+z=4,x=0, y=0, z=0

22) Use the divergence theorem to evaluate

a) $\iint_S (4x\hat{i} - 2y^2 j + z^2\hat{k}) \circ d\vec{S}$ where S is the region bounded by the surfaces

$y^2 = 4x, x = 1, z = 0, z = 3$

b) $\iint_S (x^2\hat{i} + y^2 j + z^2\hat{k}) \circ d\vec{S} = \frac{\pi}{12}$ where S is the part of the sphere $x^2 + y^2 + z^2 = 1$

lying above the xy-plane **(D-10)**

c) $\iint_S \frac{\vec{r}}{r^3} \circ d\vec{S}$ where S does/does not enclose the origin

d) $\iint_S (\phi\nabla\psi - \psi\nabla\phi) \circ d\vec{S}$ e) div curl $\vec{A}$

Stoke's Theorem

Problems

23) Verify Stoke's theorem for

a) $\vec{A} = (x + y)\hat{i} + (2x - z)\hat{j} + (y + z)\hat{k}$ over the triangle with vertices at the points (2,0,0), (0,3,0),(0,0,6) **(D-08)**

b) $\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the surface of the sphere $z = \sqrt{a^2 - x^2 - y^2}$

24) Using Stoke's theorem evaluate

a) $\iint_S \nabla \times \vec{A} \circ d\vec{S}$ where $\vec{A} = (x^2 + y - 4)\hat{i} = 3xy\hat{j} + (2xz + z^2)\hat{k}$ and S is the surface above the xy-plane of the paraboloid $z = 4 - (x^2 + y^2)$

b) $\iint_S \nabla \times \vec{A} \circ d\vec{S}$ where S is the part of the surface $x^2 + y^2 + z^2 - 2ax + az = 0$ and

$\vec{A} = (2y^2 + 3z^2 - x^2)\hat{i} + (2z^2 + 3x^2 - y^2)\hat{j} + (2x^2 + 3y^2 - z^2)\hat{k}$

25) Verify Stoke's theorem for

a) $\vec{A} = \sin z\hat{i} - \cos x\hat{j} + \sin y\hat{k}$ where C is the boundary of the rectangle $0 \le x \le \pi, 0 \le y \le 1, z = 3$

b) $\vec{A} = y\hat{i} + z\hat{j} + x\hat{k}$ over the surface $x^2 + y^2 = 1 - z \ \ (z > 0)$ **(D-09)**

26)Using stoke's theorem evaluate $\oint_C ydx + zdy + xdz$ where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and the plane x + z=a **(D-10)**

LAPLACE TRANSFORM

Problems

01) Using the definition , find the Laplace Transform of the following functions

(a) $F(t) = \begin{cases} t & 0 < t < 4 \\ 5 & t > 4 \end{cases}$ (b) $F(t) = \begin{cases} \sin t & 0 < t < \pi \\ \cos t & t > \pi \end{cases}$

02) Using the definition , find the Laplace Transform of the following functions

(a) $F(t) = \begin{cases} 2(t-1)^2 & 0 < t < 5 \\ 1 & t > 5 \end{cases}$ (b) $F(t) = \begin{cases} \cos t & 0 < t < 2\pi \\ 0 & t > 2\pi \end{cases}$

Problems

03) Find the Laplace Transform of following functions

(a) $(t+1)^3$ (b) $\cos^4 t$ (c) $\sin 2t \sin 4t \sin 6t$

04) Show that

$(a)\, L[\sin\sqrt{t}] = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-1/4s}$

$(b)\, L\{\sin^3 t\} = \frac{3!}{(s^2+1)(s^2+9)}$ and hence show that $\int_0^\infty e^{-2t} \sin^3 t \; dt = \frac{3}{65}$

$(c)\alpha = \frac{\pi}{4}$ Using Laplace Transform if $\int_0^\infty e^{-2t} \sin(t+\alpha)\cos(t-\alpha)dt = \frac{3}{8}$

05) Find the Laplace Transform of following functions

(a) $\left(\sqrt{t}-1\right)^4$ (b) cos2tcos4tcos6t (c) $\cosh^4 t$ **(D-07)**

(d) $\sin t^2$ (e) $\sin^5 t$ (f) $\sinh^2 4t$

06) Show that

$(a)\, L[\frac{\cos\sqrt{t}}{\sqrt{t}}] = \frac{\sqrt{\pi}}{\sqrt{s}} e^{-1/4s}$ $(b)\, L\{\sin^5 t\} = \frac{5!}{(s^2+1)(s^2+9)(s^2+25)}$

$(c)\, L[J_0(t)] = \frac{1}{\sqrt{s^2+1}}$ *and hence* $\int_0^\infty t\, e^{-3t} J_0(4t)dt = \frac{3}{125}$ *if* $J_0(t) = \sum_0^\infty \frac{(-1)^r}{(r!)^2}\left(\frac{t}{2}\right)^{2r}$

First Shift Theorem

Problems

07) Find the Laplace transform of the following functions

(a) $(1+te^{-t})^3$ (b) $t^5 \sinh t$ (c) $e^{-2t}\sin^2 4t$ (d) $(t^2 \sinh t)^2$

08) Find the Laplace transform of the following functions

(a) $e^{-3t}\sin 3t \cosh 2t$ (b) $\sinh at \cos at$ (c) $\left(\frac{\cos t + \sin t}{e^t}\right)^2$

Second Shift Theorem

Problem

09) *Find* $L\{G(t)\}$ *where* $G(t) = 0$ *for* $0 < t < \frac{2\pi}{3}$ *and* $\cos(t - \frac{2\pi}{3})$ *for* $t > \frac{2\pi}{3}$

10) Find the Laplace transform of $(t-1)^2 u(t-1)$ and $e^{-3t}u(t-2)$ where $u(t-a)=\begin{cases}0; t<a\\ 1; t\geq a\end{cases}$ is the unit step function **(M-11)**

11) *Find* $L\{G(t)\}$ *where* $G(t)=0$ *for* $0<t<\frac{2\pi}{3}$ *and* $\sin^2(t-\frac{2\pi}{3})$ *for* $t>\frac{2\pi}{3}$

Change of Scale Theorem

Problem

12)Find $L\{F(3t)\}$ and $L\{F(\frac{t}{2})\}$ if given $L\{F(t)\}=\frac{1-3s}{s^2-4s+2}$

13) Find $L\{e^{-t}F(2t)\}$ if given $L[F(t)]=\frac{1}{s(s^2+1)}$

Multiplication By t Theorem

Problems

14) Find the Laplace transform of the following functions and hence evaluate the given integral

(a) $t\sin^2 t$; $\int_0^\infty e^{-2t}t\sin^2 t\,dt=\frac{1}{8}$ (b) $t\sqrt{1+\sin t}$; $\int_0^\infty e^{-t}t\sqrt{1+\sin t}\,dt=\frac{28}{25}$

(c) $t^2\sin\sqrt{3}t$; $\int_0^\infty e^{-t}t^2\sin\sqrt{3}\,t\,dt=0$ (d) $t^3\sin t$; $\int_0^\infty e^{-t}t^3\sin t\,dt=0$ **(M-10)**

15) Find the Laplace transform of the following functions

(a) $te^{-2t}\sin(at-b)$ **(D-08)** (b) $t^2\sin at$ **(M-11)** (c) $(t\sinh 2t)^2$ **(M-10)**

(d) $t^2\sin^2 2t$ (≅ −08`) (e) $\frac{\sqrt{1+\sin 4t}}{e^{2t}}$ (≅ −08`)

(*f*) $te^{3t}\cos 2t$;and hence show that $\int_0^\infty e^{3t}t\cos 2t\,dt=\frac{5}{169}$

(g) $\int_0^\infty \cos(tx^2)dx$ *and hence evaluate* $\int_0^\infty \cos x^2\,dx$ (≅ −10`)

Division By t Theorem

Problems

15) Find the Laplace transform of the following functions & hence evaluate the integral

$(a)\frac{\sin^2 t}{t};\int_0^\infty e^t\frac{\sin^2 t}{t}dt=\frac{1}{4}\log 5$ (b) $\frac{\sin 2t+\sin 3t}{t};\int_0^\infty\frac{\sin 2t+\sin 3t}{te^t}dt=\frac{3\pi}{4}$

(c) $\frac{e^{-at}-e^{-bt}}{t};\int_0^\infty\left(\frac{e^{-3t}-e^{-6t}}{t}\right)dt=\log 2$

$(d)\frac{\cos at-\cos bt}{t};\int_0^\infty\left(\frac{\cos 6t-\cos 4t}{t}\right)dt=\log\frac{2}{3}$ $(d)\frac{\cos at-\cos bt}{t}$

(e) $\frac{\sin t\sinh t}{t};\int_0^\infty e^{-\sqrt{2}t}\frac{\sin t\sinh t}{t}dt=\frac{\pi}{8}$

16) Find the Laplace transform of the following functions & hence evaluate the integral

(a) $\frac{\sin t}{t};\int_0^\infty\frac{\sin t}{t}dt=\frac{\pi}{2}$ (b) $\frac{\sin t\sin 6t}{t}$ (c) $\frac{e^{-4t}\sin 3t}{t}$ (M – 09`

$(d)\frac{\sin 2t\cosh 2t}{t}$ $(e)\frac{\cosh 2t-\sin 2t}{t}$ (M – 10`

Laplace Transform Of Integral

Problems

17) Find Laplace Transform of the following functions

$(a)\int_0^t e^{-u}\frac{\sin 4u}{u}du$ $(b)\int_0^t\frac{1-e^{-u}}{u}du$ $(c)\int_0^t u\cos^2 u\,du$ $(d)\cosh t\int_0^t e^u\cosh u\,du$

18) Find Laplace Transform of the following functions

$(a)\int_0^t\frac{1-\cos u}{u}du$ $(b)\int_0^t ue^{-3u}\cos 4u\,du$ (c) $\int_0^t e^{-2u}\cos^2 u\,du$ $(d)\int_0^\infty\int_0^t e^{-t}\frac{\sin u}{u}du\,dt$ (M – 08`

Laplace Transform of Derivative

Problems

19)(a)Find function $L\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right)$ given $L[\sin\sqrt{t}] = \frac{\sqrt{\pi}}{2s^{3/2}}e^{-1/4s}$

(b)If $F(t) = \begin{cases} t+1 & 0 \le t \le 2 \\ 3 & t \ge 2 \end{cases}$ find $L[F'(t)]$ and $L[F''(t)]$

20) (a) If $L\{t\sin\omega t\} = \frac{2\omega}{(s^2+\omega^2)^2}$ *find* $L\{\sin\omega t + \omega t\cos\omega t\}$ **(M-11)**

(b) Find $L\left\{\frac{d}{dt}\left(\frac{\sin^2 t}{t}\right)\right\}$

Convolution Theorem

Problem

21) Verify Convolution theorem for the function $F(t) = t^2, G(t) = e^{2t}$

22) Verify Convolution theorem for the function $F(t) = \sin at, G(t) = \sin bt$

Periodic Function

Problems

23) Find the Laplace transform of the following functions with period equal to length of the given interval

$(a)\ F(t) = \frac{t}{T} \quad 0 < t < T$ $\qquad (b) F(t) = |\sin\omega t|$ $\qquad (c)\ F(t) = \begin{cases} 1 & 0 < t < a/2 \\ -1 & a/2 < t < a \end{cases}$

$(d) F(t) = \begin{cases} \frac{t}{a} & 0 < t < a \\ \frac{2a-t}{a} & a < t < 2a \end{cases}$

24) Find the Laplace transform of the following functions with period equal to length of the given interval

$(a) F(t) = \begin{cases} \sin\omega t; & 0 < t < \pi/\omega \\ 0 \quad ; & \pi/\omega < t < 2\pi/\omega \end{cases}$ $\qquad (b)\ F(t) = \begin{cases} 1; & 0 < t < 1 \\ 0; & 1 < t < 2 \end{cases}$ (M -08)

(c) F(t)=t ; 0<t<1 and 0; 1<t<2 and F(t+2)=F(t) for t>0

Heavyside's Unit Step Function

Problems

25) Prove the following results

$$(a) \quad L[F(t).H(t-a)] = e^{-as} L[F(t+a)]$$

$$(b) \quad L[H(t-a)] = \frac{e^{-as}}{s}$$

26) Find the Laplace transform of the following functions

(a) $L[t^4 H(t-1)]$

(b) $L[(1+2t-3t^2+4t^3)H(t-2)]$

28) Evaluate $\int_0^\infty e^{-t}(1+2t-t^2+t^3)H(t-1)dt$ **(M-10)**

29) Express the following function using Unit step functions and evaluate the Laplace transform

$$F(t) = \begin{cases} t^2 & 0 < t < 2 \\ 4t & t > 2 \end{cases}$$

30)

28) Prove the following results

(a) $L[F(t).H(t)] = L[F(t)] = f(s)$

(b) $L[F(t-a).H(t-a)] = e^{-as} L[F(t)]$

$(c) L[H(t)] = \frac{1}{s}$

29) Find the Laplace transform of the following functions

(a) $L[t^2 H(t-3)]$

(b) $L[(1+3t-t^2+t^3)H(t-4)]$

30) Express the following function using Unit step functions and evaluate the Laplace transform

$$(a) F(t) = \begin{cases} \sin t & 0 < t < \pi \\ \sin 2t & \pi < t < 2\pi \\ \sin 3t & t > 2\pi \end{cases}$$

$$(b) F(t) = \begin{cases} 2t & 0 < t < 1 \\ 3t^2 & t > 1 \end{cases}$$ **(M-10)**

Unit impulse(or Dirac delta) function

Problems

32) Prove the following results

$(a)\int_0^{\infty} F(t)\delta(t-a)dt = F(a)$ $(b)\, L[F(t)\delta(t-a)] = e^{-as}F(a)$

33) Find the following

(a) $L[\sin 2t\,\delta(t-\pi/4) - t^2\delta(t-4)]$ (b) $L[\cos t \log t\,\delta(t-\pi)]$

34) Prove the following results

$(a)\ L[\delta(t-a)] = e^{-as}$ $(b) L[\delta(t)] = 1$

35) Find the following $L[t\,U(t-4) - t^3\delta(t-2)]$

Error Function

Problems

36) Show that

(a) $\int_0^{\infty} e^{-t}\operatorname{erf}\sqrt{t}dt = \frac{1}{\sqrt{2}}$ (b) $L\left\{t\operatorname{erf}2\sqrt{t}\right\} = \frac{3s+8}{s^2(s+4)^{\frac{3}{2}}}$ **(D-10)**

37)Show that

(a) $L\left(\int_0^t \operatorname{erf}\sqrt{t}dt\right) = \frac{1}{s^2\sqrt{s+1}}$ (b) $L\left(e^{3t}\operatorname{erf}\sqrt{t}\right) = \frac{1}{(s-3)\sqrt{s-2}}$ (c)

$\int_0^{\infty} e^{-5t} erf\, 2\sqrt{t}dt$

INVERSE LAPLACE TRANSFORM

Problems

38)Find $\mathbf{(a)}\boldsymbol{L^{-1}}\left\{\frac{\mathbf{6}}{\mathbf{3-2}\boldsymbol{s}}-\frac{\mathbf{3+4}\boldsymbol{s}}{\mathbf{9}\boldsymbol{s^2}\mathbf{+16}}+\frac{\mathbf{8-6}\boldsymbol{s}}{\mathbf{16}\boldsymbol{s^2}\mathbf{-9}}\right\}\mathbf{(b)}\boldsymbol{L^{-1}}\left\{\frac{\mathbf{3}\boldsymbol{s}\mathbf{-2}}{\boldsymbol{s}^{\frac{\mathbf{5}}{\mathbf{2}}}}-\frac{\mathbf{3+4}\boldsymbol{s}}{\mathbf{9}\boldsymbol{s^2}\mathbf{+16}}+\frac{\mathbf{8-6}\boldsymbol{s}}{\mathbf{16}\boldsymbol{s^2}\mathbf{-9}}\right\}$

Homework

39) Find (a) $L^{-1}\left\{\left(\frac{1-\sqrt{s}}{s^2}\right)^2\right\}$(M-11) **(b)** $\mathrm{L}^{-1}\left(\frac{3s-2}{s^{5/2}}+\frac{3(s^2-1)^2}{2s^5}\right)$ **(c)** $\mathrm{L}^{-1}\left(\frac{2s+1}{s(s+1)}\right)$

Problems

40) Find the Inverse Laplace Transform of the following functions

(a) $\frac{1}{\sqrt[3]{8s-27}}$ **(b)** $\frac{6s-4}{2s^2-12s+26}$ **(c)** $\left\{\frac{1}{(s-1)^5}+\frac{3s+1}{(s+1)^4}\right\}$

41) Find the Inverse Laplace transform of the following functions

(a) $\frac{e^{-5s}}{(s+4)^3}$ (b) $\frac{8e^{-3s}}{s^2+4}$ (c) $\frac{(s+1)e^{-\pi s}}{s^2+s+1}$

42) Find the Inverse Laplace Transform of the following functions using partial fraction method

(a) $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$ (b) $\left\{\frac{\mathbf{-3}\boldsymbol{s^2}\mathbf{+20}\boldsymbol{s}\mathbf{-24}}{(\boldsymbol{s}\mathbf{-1})(\boldsymbol{s}\mathbf{-2})^{\mathbf{2}}}\right\}$ **(c)** $\frac{3s+1}{(s-1)(s^2+1)}$

(d) $\frac{s^3+2s}{(s+1)^2(s^2+1)}$ (e) $\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$ (f) $\frac{\mathbf{s}}{\mathbf{s^4+4}}$

43) Find the Inverse Laplace Transform of the following functions using convolution theorem

(a) $\frac{1}{(s^2+4)(s^2+9)}$ *(b)* $\left\{\frac{s}{s^4+13s^2+36}\right\}$ *(c)* $\frac{s^2+4s+4}{(s^2+4s+8)^2}$

(d) $\frac{1}{(s^2+a^2)^2}$ *(e)* $\frac{s}{(s^2+a^2)^2}$ *(M-11)* *(g)* $\frac{s^2}{(s^2+a^2)^2}$ *(h)* $\frac{s^3}{(s^2+a^2)^2}$

44) Find the Inverse Laplace Transform of

a) $\tan^{-1}(s+1)$ **(b)** $\tan^{-1}\frac{2}{s^2}$ **(c)** $\frac{1}{s}\log\left(\frac{s+2}{s+1}\right)$ *(d)* $\frac{1}{s}\log\sqrt{\left(\frac{s^2+a^2}{s^2+b^2}\right)}$

45) Find the Inverse Laplace Transform of the following functions

(a) $\frac{1}{\sqrt{2s+3}}$

46) Find the Inverse Laplace transform of the following functions

(a) $\frac{e^{4-3s}}{(s+4)^{5/2}}$ (b) $\frac{e^{-2s}}{s^2+8s+25}$ (c) $\frac{e^{-\Pi s}}{s^2-2s-2}$ (M -08`

47) Find the Inverse Laplace Transform of the following functions using partial fraction method

(a) $\frac{3s+7}{s^2-2s-3}$ (M -10`) (b) $\frac{s+2}{(s+1)^3(s+3)}$ (M -10`) (c) $\frac{1}{s^3+1}$

(d) $\frac{2}{(s+1)^2(s^2+4)}$ (M -08`) (e) $\frac{1}{s^3+1}$ **(f)** $\frac{2s^3+10s^2+8s+40}{s^2(s^2+9)}$

(g) $\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$ **(h)** $\frac{s}{s^4+4}$ **(i)** $\frac{s}{s^4+s^2+1}$

(j) $\frac{2s^2-1}{(s^2+1)(s^2+4)}$ **(M-11)** **(k)** $\frac{s}{(s^2+1)(s^2+4)(s^2+9)}$ **(l)** $\frac{6s+3}{s^4+5s^2+4}$

48) Find the Inverse Laplace Transform of the following functions using convolution theorem

(a) $\frac{s^2}{(s^2+4)(s^2+9)}$ *(b)* $\frac{s}{(s^2+4)^2}$ (≅ −08` *(c)* $\frac{1}{(s^2-a^2)^2}$ (≅ −08`

$(d)\frac{1}{(s-4)^4(s+3)}$ *(e)* $\frac{s}{(s^2-a^2)^2}$ *(f)* $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ *(g)* $\frac{s^2+5}{(s^2+4s+13)^2}$

49) Find the Inverse Laplace Transform of

(a) $\tan^{-1}(s+1)$ **(b)** $\cot^{-1}\frac{s-2}{3}$

(c) $\tan^{-1}\frac{2}{s^2}$ $(d)\frac{1}{s}\log(1+\frac{1}{s^2})$

Application of Laplace Transform

Problems

50) Solve the following equations

(a) $y''+2y'+5y = e^{-t}\sin t$; $y(0)=0, y'(0)=1$ **(M-11)**

(b) $y'+2y+\int_0^t ydt = \sin t$; $y(0)=1$ **(M-08)**

(c) $y''+9y = \cos 2t$; $y(0)=1, y(\pi/2) = -1$

(d) $y''-3y'+2y = 4e^{2t}$; $y(0)= -3, y'(0)=5$

(e) $y''+4y = f(t)$, $y(0)=0, y'(0)=1$ where f(t)=1 when 0<t<1 and f(t)=0 when t>1

(a) $y''-3y'+2y = 4e^{2t}$; $y(0)= -3, y'(0)=5$

(b) $y''-y'-2y = 20\sin t$; $y(0)=1, y'(0)=2$ **(M-10)**

(c) $y+\int_0^t ydt = 1-e^{-t}$

(d) $y''+9y = 18t$; $y(0)=0,\ y(\pi/2) = 0$

(e) $y'''-3y''+3y'-y = t^2e^t$; $y(0)=1, y'(0)=0, y''(0)=-2$

(f) $y''+3y'+2y = t\,\delta(t-1)$; $y(0)= 0, y'(0)=1$

FOURIER SERIES

Neither Even nor Odd Functions

Problems

01. Find the Fourier series expansion of the functions in the respective intervals

(a) $f(x) = x^2$, $0 < x < 2$ and hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$ **(M-09)**

and $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$ **(D-09)**

(b) $f(x) = 4 - x^2, 0 < x < 2$ & hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots. = \frac{\pi^2}{6}$

(c) $f(x) = x \sin x$; $0 \le x \le 2\pi$

(d) $f(x) = \begin{cases} 0 & ; -\pi \le x \le 0 \\ \sin x & ; 0 \le x \le \pi \end{cases}$ and hence deduce that

(i) $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} \ldots. = \frac{1}{2}$ (ii) $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} \ldots. = \frac{\pi - 2}{4}$ **(D-07)**

(iii) $\frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + \ldots. = \frac{\pi}{8}$

02. Find the Fourier series expansion of the functions in the respective intervals

(a) $f(x) = \frac{\pi - x}{2}, x\varepsilon(0, 2\pi)$ & hence deduce that $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots. = \frac{\pi}{4}$ **(M-11)**

(b) $f(x) = \cos ax$; $0 < x < 2\pi$; $a \neq$ integer and hence deduce that

$\pi \cot a\pi = \frac{1}{a} + 2a \sum_{n=1}^{\infty}\frac{1}{a^2 - n^2}$ & $\pi \operatorname{cosec} a\pi = \frac{1}{a} + 2a \sum_{n=1}^{\infty}\frac{(-1)^n}{a^2 - n^2}$

(c) $f(x) = \begin{cases} 0 & ; -\pi < x < 0 \\ x^2 & ; 0 < x < \pi \end{cases}$ where f(x) is periodic with 2π **(M-10, M-09)**

(d) $f(x) = \begin{cases} 2 & ; -2 < x < 0 \\ x & ; 0 < x < 2 \end{cases}$ (Ω −09`

Even Functions

Problems

03. Find the Fourier expansion for the following functions

(a) $f(x) = x^2$ in $(-\pi,\pi)$ and hence deduce that **(M-09,D-10)**

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots.=\frac{\pi^2}{6} \qquad \frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\ldots.=\frac{\pi^2}{12}$$

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots.=\frac{\pi^2}{8} \qquad \frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots.=\frac{\pi^4}{90}$$

(b) $f(x) = \frac{\pi^2}{12}-\frac{x^2}{4}$ in $(-\pi,\pi)$ **(M-11)**

(c) $f(x) = |\sin x|$ **(D-07)**

(d) $f(x) = x \sin x$ in $(-\pi,\pi)$ & deduce $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}\ldots. = \frac{\pi-2}{4}$

(d) $f(x)=\begin{cases}1+\frac{2x}{\pi} ; & -\pi \le x \le 0\\ 1-\frac{2x}{\pi} ; & 0 \le x \le \pi\end{cases}$ & deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots.=\frac{\pi^2}{8}$ **(D-08)**

04. Find the Fourier series expansion for the following functions

(a) $f(x) = \left(\frac{\pi-x}{2}\right)^2$ in $(0,2\pi)$ and hence deduce that

$(i)\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots.=\frac{\pi^2}{6}$ $(ii)\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\ldots.=\frac{\pi^2}{12}$ **(M-08,D-10)**

(b) $f(x) = \sqrt{1-\cos x}, o < x < 2\pi$ & deduce that $\sum_{n=1}^{\infty}\frac{1}{\left(4n^2-1\right)}=\frac{1}{2}$ **(M-10,D-09)**

(c) $f(x)=\begin{cases}\frac{1}{2}+x\ ; & -\frac{1}{2}<x\le 0\\ \frac{1}{2}-x\ ; & 0<x\le\frac{1}{2}\end{cases}$, f(x) is a periodic function of period 1. **(D-08)**

(d) $f(x)=\begin{cases}x+\frac{\pi}{2}\ ; & -\pi<x<0\\ \frac{\pi}{2}-x\ ; & 0<x<\pi\end{cases}$ **(D-09)**

(e) $f(x)=\begin{cases}x\ ; & 0<x<\frac{\pi}{2}\\ \pi-x\ ; & \frac{\pi}{2}<x<\pi\end{cases}$ & hence deduce that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4}=\frac{\pi^4}{96}$ **(D-07, M-11)**

Odd Functions

Problems

05. Find the Fourier expansion for the following functions

(a) $f(x)=\begin{cases}-(\pi+x) & -\pi\le x\le -\pi/2\\ x & -\pi/2\le x\le \pi/2\\ \pi-x & \pi/2\le x\le\pi\end{cases}$

(b)Prove that in the interval $0<x<\pi$,

$$\frac{e^{ax}-e^{-ax}}{e^{a\pi}-e^{-a\pi}}=\frac{2}{\pi}\left[\frac{1\sin x}{a^2+1^2}-\frac{2\sin 2x}{a^2+2^2}+\frac{3\sin 3x}{a^2+3^2}-.....\right]$$ **(D-08)**

(c)f (x) = x cos x in $(-\pi,\pi)$ **(M-08)**

(d) $f(x)=x-x^2$,-1< x < 1 **(D-09)**

(e) $f(x)=x^3\ in\ (-\pi\ ,\ \pi)$ **(M-09)**

06. Find the Fourier expansion for the following functions

(a) $f(x)=\begin{cases}\pi x & ;\ 0<x<1\\ \pi(x-2) & ;\ 1<x<2\end{cases}$ **(D-07)** & hence S.T $1-\frac{1}{3}+\frac{1}{5}-...=\frac{\pi}{4}$

(b) $f(x)=\begin{cases}x & ;\ 0\le x<\pi/2\\ \pi-x & ;\ \pi/2\le x\le 3\pi/2\\ x-2\pi & ;\ 3\pi/2\le x\le 2\pi\end{cases}$

(c) $f(x) = x|x|$ *in* $(-1,1)$ **(M-10,D-09)**

(d) $f(x) = x^2$ in $(-\pi, \pi)$ **(M-09)**

(e) $f(x) = 2x - x^2, 0 < x < 2$ **(M-08)**

Half Range Fourier Series

Problems

07. Find the half range Fourier sine /cosine series of the function

(a) $f(x) = x$; $0 < x < 2$ **(D-09)** and hence deduce that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{94} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

(b) $f(x) = \begin{cases} kx & ; 0 \le x \le L/2 \\ k(L-x) & ; L/2 \le x \le L \end{cases}$ and hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8} \text{ and } \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots = \frac{\pi^2}{96}$$

(c) $f(x) = x(L-x)$; $0<x<L$ and hence deduce that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^3} = \frac{\pi^3}{32}$ **(M-11)**

(d) $f(x) = \begin{cases} 1 & ; 0 < x < 1 \\ x & ; 1 < x < 2 \end{cases}$ **(M-08)**

(e) Find the half range cosine series for $f(x) = \sin x$; $0 \le x \le \pi$ and hence deduce that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} \ldots = \frac{1}{2} \quad , \quad \frac{1}{1^2.3^2} + \frac{1}{3^2.5^2} + \frac{1}{5^2.7^2} \ldots = \frac{\pi^2 - 8}{16}$$ **(D-10)**

08. Find the half range Fourier sine /cosine series of the function

(a) Find the half range Fourier cosine series of $f(x) = x(\pi - x)$; $0 < x < \pi$ and hence deduce that **(D-09)**

(i) $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ (D -09,

(ii) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^2}=\frac{\pi^2}{12}$ (D -09`

(iii) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^3}=\frac{\pi^3}{32}$,

(iv) $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4}=\frac{\pi^4}{96}$, $\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$

(b) f(x)=x sin x in [0, π] and hence deduce that

$$\frac{1}{1^2}-\frac{1}{3^2}-\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{9^2}-\frac{1}{11^2}-\frac{1}{13^2}+\ldots\ldots\ldots=\frac{\pi^2}{8\sqrt{2}}$$

(c) $f(x)=\frac{\pi}{4}$ in (0, π) and hence deduce that **(M-10)**

(i) $\frac{\pi}{4}\left(\frac{\pi}{2}-x\right)=\frac{1}{1^2}\cos x+\frac{1}{3^2}\cos 3x+\frac{1}{5^2}\cos 5x+\ldots..$ and

(ii) $\frac{\pi}{8}x(\pi-x)=\frac{1}{1^3}\sin x+\frac{1}{3^3}\sin 3x+\frac{1}{5^3}\sin 5x+\ldots..$

(d) Find the half range cosine series of f(x)= sin $\frac{\pi x}{L}$; 0 < x < L **(M-09)**

Complex Form of Fourier series

Problems

09. Find the complex form of Fourier series of following functions in the respective intervals(a) $f(x)=e^{-x}, -1<x<1$ (b) f (x) = cosh ax ,(L,-L) **(D-09,D-**

(c)f(x)=cosax , $-\pi<x<\pi$ ($a\neq$ integer) **(M-09,M-11)**

Homework

(a) $f(x)=e^{ax}, -L<x<L$ (M -08,D-07`

(b) f (x) = 2x , $0<x<2\pi$ **(D-09)**

(c) $f(x)=\begin{cases}0; & 0<x<L\\ a; & L<x<2L\end{cases}$ (D -08`

Orthogonal and Orthonormal Functions

Problems

10. Determine if the following set of functions are orthogonal or orthonormal , and find the corresponding set of orthonormal functions in the case of orthogonal functions.

(a) $x, \frac{1}{2}(3x^2-1), \frac{1}{2}(5x^3-3x)$ in (-1 , 1)

(b) cos x , cos 2x , cos 3x ,. . . . .in [0, π] **(M-08)**

(c) sin x , sin 3x , sin 5x ,. . . . . in [0 , $\frac{\pi}{2}$] **(D-09,D-08,D-07,D-10)**

(d) $1, \sin\frac{\pi x}{T}, \cos\frac{\pi x}{T}, \sin, \cos\frac{2\pi x}{T}, \ldots\ldots [0, 2T]$

11. If the functions $x, \frac{1}{2}(ax^2-1), \frac{1}{2}(bx^3-3x)$ are orthogonal find a and b

12. Show that the functions $1, x, \frac{1}{2}(3x^2-1)$ are orthogonal and find the corresponding set of orthonormal functions.

13. Show that the following set of functions is orthonormal.

$e^{\frac{-x}{2}}, e^{\frac{-x}{2}}(1-x), \frac{1}{2}e^{\frac{-x}{2}}(2-4x+x^2)$ in $(0, \infty)$

14. Show that the following set of functions $\sin\frac{(2n+1)\pi x}{L}$, n=0,1,2,… is orthogonal over [0 , L].Hence construct an orthonormal set of functions.**(M-11)**

15. Define orthogonal and orthonormal set of functions. S.T. $\{\sin nx\}_{n=1,2,3,\ldots\ldots}$ is orthogonal set of functions over [0, π].Hence construct orthonormal set of functions.

Fourier Integral

Problems

01. Find the Fourier integral of the function

(a) $f(x)=\begin{cases}0 & , x<0\\ e^{-x} & , x>0\end{cases}$ & deduce that $\int_0^\infty \frac{\cos\lambda x+\lambda\sin\lambda x}{\lambda^2+1}d\lambda=\begin{cases}0 & ;x<0\\ \frac{\pi}{2} & ;x=0\\ \pi e^{-x} & ;x>0\end{cases}$

(b) $f(x)=\begin{cases}1 & , |x|<1\\ 0 & , |x|>1\end{cases}$ & hence show that $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda=\begin{cases}\frac{\pi}{2} & ;|x|<1\\ \frac{\pi}{4} & ;|x|=1\\ 0 & ;|x|>1\end{cases}$ **(M-11)**

02. Using the Fourier integral representation show that

$$\int_0^{\infty} \frac{\cos\lambda x + \lambda \sin\lambda x}{1+\lambda^2} d\lambda = \begin{cases} 0, & x < 0 \\ \frac{\pi}{2}, & x = 0 \\ \pi e^{-x}, & x > 0 \end{cases}$$

03. Find the Fourier integral representation of the function $f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}, & x = 0 \\ e^{-x}, & x > 0 \end{cases}$ **(M-10,D-10)k**

04. Find the Fourier integral of the function

(a) $f(x) = \begin{cases} -e^{kx}, & x < 0 \\ e^{-kx}, & x > 0 \end{cases}$ & hence S.T. $\int_0^{\infty} \frac{\lambda \sin\lambda x}{\lambda^2 + k^2} d\lambda = \frac{\pi}{2} e^{-kx}$ for x>0,k>0

(b) $f(x) = \begin{cases} e^{ax}, & x \le 0 \\ e^{-ax}, & x \ge 0 \end{cases}$ **(D-09)** & hence S.T. $\int_0^{\infty} \frac{\cos\lambda x}{\lambda^2 + a^2} d\lambda = \frac{\pi}{2a} e^{-ax}$; x > 0, a > 0

05. Express the function $f(x) = \begin{cases} 1 \text{ for } |x| < 1 \\ 0 \text{ for } |x| > 1 \end{cases}$ as Fourier integral, and hence evaluate $\int_0^{\infty} \frac{\sin\omega \cos\omega x}{\omega} d\omega$, $\int_0^{\infty} \frac{\sin\omega x}{\cos\omega} d\omega$ and $\int_0^{\infty} \frac{\sin\omega}{\omega} d\omega$

06. If $f(x) = \begin{cases} \sin x & \text{when } 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$ then show that $f(x) = \frac{1}{\pi} \int \frac{\cos\lambda x + \cos\lambda(\pi - x)}{1-\lambda^2} d\lambda$ and hence deduce that $\int_0^{\infty} \frac{\cos\frac{\lambda\pi}{2}}{1-\lambda^2} d\lambda = \frac{\pi}{2}$

Vector Algebra and Vector Calculus

Vector Algebra

1) Prove that $\bar{d} \cdot \left[\bar{a} \times \left\langle \bar{b} \times (c \times d)\right\rangle\right] = \left(\bar{b} \cdot \bar{d}\right)\left[\bar{a}\ \bar{c}\ \bar{d}\right]$

2) Prove that $(a \times b) \cdot (c \times d) = \begin{vmatrix} \bar{a} \cdot \bar{c} & \bar{a} \cdot \bar{d} \\ \bar{b} \cdot \bar{c} & \bar{b} \cdot \bar{d} \end{vmatrix}$ & hence deduce that

$(b \times c) \cdot (a \times d) + (c \times a) \cdot (b \times d) + (a \times b) \cdot (c \times d) = 0$

3) Prove that

(i) $\left(\bar{a} \times \bar{b}\right) \times \left(\bar{c} \times \bar{d}\right) = \left[\bar{a}\bar{b}\bar{c}\right]\bar{c} - \left[\bar{a}\bar{b}\bar{c}\right]\bar{d}$

(ii) $\left(\bar{a} \times \bar{b}\right) \times \left(\bar{c} \times \bar{d}\right) = \left[\bar{a}\bar{c}\bar{d}\right]\bar{b} - \left[\bar{b}\bar{c}\bar{d}\right]\bar{a}$

4) Expand $\left(\bar{a} \times \bar{b}\right) \times \left(\bar{c} \times \bar{d}\right)$ in two different ways & deduce that

$$\bar{d} = \frac{\left[\bar{b}\ \bar{c}\ \bar{d}\right]a + \left[\bar{c}\ \bar{a}\ \bar{d}\right]\bar{b} + \left[\bar{a}\ \bar{b}\ \bar{d}\right]\bar{c}}{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}$$

5) Prove that the three vectors $\bar{a} \times \left(\bar{b} \times \bar{c}\right), \bar{b} \times (\bar{c} \times \bar{a}), \bar{c} \times \left(\bar{a} \times \bar{b}\right)$ are coplanar.

6) If $a \neq 0, a \cdot b = a \cdot c$ and $\bar{a} \times \bar{b} = \bar{a} \times \bar{c}$, then prove that $\bar{b} = \bar{c}$

7) Prove that $\left[\left(\bar{a} + \bar{b} + \bar{c}\right) \times \left(\bar{b} + \bar{c}\right)\right] \cdot \bar{c} = \left[\bar{a}\ \bar{b}\ \bar{c}\right]$

8) Prove that $\hat{i} \times \left(\bar{a} \times \hat{i}\right) + \hat{j} \times \left(\bar{a} \times \hat{j}\right) + \hat{k} \times \left(\bar{a} \times \hat{k}\right) = 2\bar{a}$

9) Prove that $\begin{vmatrix} \bar{p} & \bar{q} & \bar{r} \\ \bar{a} \cdot \bar{p} & \bar{a} \cdot \bar{q} & \bar{a} \cdot \bar{r} \\ \bar{b} \cdot \bar{p} & \bar{b} \cdot \bar{q} & \bar{b} \cdot \bar{r} \end{vmatrix} = \left[\bar{p}\ \bar{q}\ \bar{r}\right]\left(\bar{a} \times \bar{b}\right)$

10) Prove that $\left[\bar{a}\ \bar{b}\ \bar{c}\right]^2 = \begin{vmatrix} \bar{a} \cdot \bar{a} & \bar{a} \cdot \bar{b} & \bar{a} \cdot \bar{c} \\ \bar{b} \cdot \bar{a} & \bar{b} \cdot \bar{b} & \bar{b} \cdot \bar{c} \\ \bar{c} \cdot \bar{a} & \bar{c} \cdot \bar{b} & \bar{c} \cdot \bar{c} \end{vmatrix}$

11) Prove that $\bar{a} \times \left[\bar{b} \times \left(\bar{c} \times \bar{d}\right)\right] = \left(\bar{b} \cdot \bar{d}\right)\left(\bar{a} \times \bar{c}\right) - \left(\bar{b} \cdot \bar{c}\right)\left(\bar{a} \times \bar{d}\right)$

12) Prove that $\left(\bar{a} \times \bar{b}\right) \times \left(\bar{a} \times \bar{c}\right) \cdot \bar{d} = \left(\bar{a} \cdot \bar{d}\right)\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

13) Attempt the following :

(i) If $\bar{a} \times \left(\bar{b} \times \bar{c}\right) = \left(\bar{a} \times \bar{b}\right) \times \bar{c}$, show that $\left(\bar{a} \times \bar{c}\right) \times \bar{b} = 0$

(ii) If $\bar{a} + \bar{b} + \bar{c} = 0$ prove that $\bar{a} \times \bar{b} = \bar{b} \times \bar{c} = \bar{c} \times \bar{a}$

14) Prove that $\left(\bar{a} \times \bar{b}\right) \cdot \left[\left(\bar{b} \times \bar{c}\right) \times \left(\bar{c} \times \bar{a}\right)\right] = \left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$

15) Prove that $\left(\bar{a} \times \bar{b}\right) \cdot \left(\bar{c} \times \bar{d}\right) + \left(\bar{b} \times \bar{c}\right) \cdot \left(\bar{a} \times \bar{d}\right) + \left(\bar{c} \times \bar{a}\right) \cdot \left(\bar{b} \times \bar{d}\right) = 0$

16) If the vector $\bar{x}$ and the scalar λ satisfy the equations $\bar{a} \times \bar{x} = \lambda\bar{a} + \bar{b}$ and $\bar{a} \cdot \bar{x} = 1$. Find the values of λ and $\bar{x}$ in terms of $\bar{a}$ and $\bar{b}$. Determine them if $\bar{a} = \hat{i} - 2\hat{j}$ and $\bar{b} = 2\hat{i} + \hat{j} - 2\hat{k}$.

17) If $\bar{r}$ is coplanar with $\bar{a}$ and $\bar{b}$, then show that $\bar{r} = \dfrac{\begin{vmatrix} \bar{r} \cdot \bar{a} & \bar{a} \cdot \bar{b} \\ \bar{r} \cdot \bar{b} & \bar{b} \cdot \bar{b} \end{vmatrix}\bar{a} - \begin{vmatrix} \bar{r} \cdot \bar{a} & \bar{a} \cdot \bar{a} \\ \bar{r} \cdot \bar{b} & \bar{a} \cdot \bar{b} \end{vmatrix}\bar{b}}{(\bar{a})^2(\bar{b})^2 - \left(\bar{a} \cdot \bar{b}\right)^2}$

18) If the vectors $\bar{u}, \bar{v}, \bar{w}$ are non–coplanar show that the vectors $\bar{u} \times \bar{v}, \bar{v} \times \bar{w}, \bar{w} \times \bar{u}$ are also non–coplanar. Hence obtain the scalars *l*, *m*, *n* such that

$\bar{u} = l(\bar{v} \times \bar{w}) + m(\bar{w} \times \bar{u}) + n(\bar{u} \times \bar{v})$

19) If $\bar{a}, \bar{b}, \bar{c}$ are three vectors defined by $\bar{a} = \frac{\bar{q} \times \bar{r}}{[\bar{p}\ \bar{q}\ \bar{r}]}, \bar{b} = \frac{\bar{r} \times \bar{p}}{[\bar{p}\ \bar{q}\ \bar{r}]}, \bar{c} = \frac{\bar{p} \times \bar{q}}{[\bar{p}\ \bar{q}\ \bar{r}]}$ then prove that $\bar{p} \times \bar{a} + \bar{q} \times \bar{b} + \bar{r} \times \bar{c} = 0$.

20) Solve simultaneously, $\bar{r} \times \bar{b} = \bar{a} \times \bar{b}$ & $\bar{r} \cdot \bar{a} = 0$ where $\bar{a} \cdot \bar{b} \neq 0$.

21) If $\bar{A} = \hat{i} + 2\hat{j} - 3\hat{k}, \bar{B} = 2\hat{i} + \hat{j} - \hat{k}, C = \hat{i} + 8\hat{j} - 2\hat{k}$, Find $\left|\bar{A} \times (\bar{B} \times \bar{C})\right|$.

22) If $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \bar{b} = -2\hat{i} + \hat{j} + \hat{k}, \bar{C} = 10\hat{j} - \hat{k}$ then determine u, v, w such that $(\bar{a} \times \bar{b}) \times \bar{c} = u\bar{a} + v\bar{b} + w\bar{c}$

23) Prove that $\bar{a} \times [\bar{a} \times (\bar{a} \times \bar{b})] \cdot \bar{c} = -(\bar{a} \cdot \bar{a})[\bar{a}\bar{b}\bar{c}]$

24) Find λ such that the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ coplanar.

25) Find the scalars p and q if $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$ where

$\bar{a} = 2\hat{i} + \hat{j} + p\hat{k}, \bar{b} = \hat{i} - \hat{j}$

$\bar{c} = 4\hat{i} + q\hat{j} + 2\hat{k}$

26) Show that the vectors $(\bar{b} \times \bar{c}), (\bar{c} \times \bar{a}), (\bar{a} \times \bar{b})$ are coplanar of a, b, c are co-planar.

27) Prove that the 4 points $4\hat{i} + 5\hat{j} + \hat{k}, -(\hat{j} + \hat{k}), 3\hat{i} + 9\hat{j} + 4\hat{k}, 4(-\hat{i} + \hat{j} + \hat{k})$ are coplanar.

Vector Calculus

1) If $\bar{r} = \bar{a}\, e^{2t} + \bar{b}\, e^{-2t}$, Show that $\frac{d^2\bar{r}}{dt^2} - 4\bar{r} = 0$.

2) If $\bar{r} = 4a\sin^2\theta\hat{i} + 4a\cos^2\theta\hat{j} + 3b\cos 2\theta\hat{k}$,

(i) $\left|\frac{d\bar{r}}{d\theta} \times \frac{d^2\bar{r}}{d\theta^2}\right|$ (ii) $\left[\frac{d\bar{r}}{d\theta}\ \frac{d^2\bar{r}}{d\theta^2}\ \frac{d^3\bar{r}}{d\theta^3}\right]$

3) Evaluate (i) $\frac{d}{dt}\left[\left(r \times \frac{d\bar{r}}{dt}\right) \times \frac{d^2\bar{r}}{dt^2}\right]$

(ii) $\frac{d^2}{dt^2}\left[\bar{r}\ \frac{d\bar{r}}{dt}\ \frac{d^2\bar{r}}{dt^2}\right]$ where $\bar{r}$ is a vector function of 't'

4) If $\bar{r} = \bar{a}\sinh t + \bar{b}\cosh t$, where $\bar{a}$ and $\bar{b}$ are constant vectors prove that

(i) $\frac{d^2\bar{r}}{dt^2} = \bar{r}$

(ii) $\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2} =$ constant.

5) Find $\frac{d}{d\theta}[\bar{a} \times (\bar{b} \times \bar{c})]$ at $\theta = \frac{\pi}{2}$

For $\bar{a} = \sin\theta\hat{i} + \cos\theta\hat{j} + \theta\hat{k}$, $\bar{b} = \cos\theta\hat{i} - \sin\theta\hat{j} - 3\hat{k}$, $\bar{c} = 2\hat{i} + 3\hat{j} - 3\hat{k}$

6) If $\bar{r} = \bar{a}\cos\omega t + \bar{b}\sin\omega t$, then prove that

(i) $\bar{r} \times \frac{d\bar{r}}{dt} = \omega(\bar{a} \times \bar{b})$

(ii) $\frac{d^2\bar{r}}{dt^2} = -\omega^2\bar{r}$

7) If $\bar{r} = \bar{a}\sin\omega t - \bar{b}\sin\omega t + \frac{\bar{c}t}{\omega^2}\sin\omega t$, prove that $\frac{d^2\bar{r}}{dt^2} + \omega^2\bar{r} = \frac{2\bar{c}}{\omega}\cos\omega t$.

8) If $\bar{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + at\tan\alpha\hat{k}$

(i) Find $\left|\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2}\right|$ **(Dec – 10)**

(ii) Prove that $\left[\frac{dr}{dt}\ \frac{d^2r}{dt^2}\ \frac{d^3r}{dt^3}\right] = a^3\tan\alpha$. **(May – 08)**

9) If $\frac{d\bar{a}}{dt} = \bar{u} \times \bar{a}$ & $\frac{d\bar{b}}{dt} = \bar{u} \times \bar{b}$, show that $\frac{d}{dt}(\bar{a} \times \bar{b}) = \bar{u} \times (\bar{a} \times \bar{b})$ **(May – 09)**

10) Prove that $\frac{d}{dt}\left[\bar{v}\ \frac{d\bar{v}}{dt}\ \frac{d^2\bar{v}}{dt^2}\right] = \left[\bar{v}\ \frac{d\bar{v}}{dt}\ \frac{d^3\bar{v}}{dt^3}\right]$

11) Prove that $\left|\bar{f} \times \frac{d\bar{f}}{dt}\right| = \left|\frac{d\bar{f}}{dt}\right|$ where $\bar{f}$ is an unit vector.

12) Find $\frac{d}{dt}\left(\frac{\bar{r} \times \bar{a}}{\bar{r} \cdot \bar{a}}\right)$ where $\bar{r}$ is a vector function of scalar t and $\bar{a}$ is a constant vector.

13) Write down the formula for $\frac{d}{dt}(\bar{A} \times \bar{B})$ and verity the same for $\bar{A} = 5t^2\hat{i} + t\,\hat{j} - 3\,t^3\hat{k}$ and $\bar{B} = \sin t\,\hat{i} - \cos t\,\hat{j}$

14) If $\bar{r} = t^3\hat{i} + \left(2t^3 - \frac{1}{5t^2}\right)\hat{j}$, then show that $\bar{r} \times \frac{d\bar{r}}{dt} = \hat{k}$

15) Find the magnitude of the velocity and acceleration of a particle which moves along the curve x = 2sin 3t, y = 2 cos3t, z = 8t at any time $t > 0$ Find unit tangent vector to the curve.

16) A particle moves along a plane curve such that its linear velocity is perpendicular to the radius vector. Show that the path of the particle is a circle.

17) If $\bar{A} = (\sin t)\hat{i} + (\cos t)\,\hat{j} + t\hat{k}$, $\bar{B} = (\cos t)\hat{i} - (\sin t)\,\hat{j} - 3\hat{k}$, $\bar{C} = 2\hat{i} + 3\hat{j} - \hat{k}$.

Find $\frac{d}{2t}\left(\bar{A} \times \left(\bar{B} \times \bar{C}\right)\right)$ at $t = 0$

Ans:- $7\hat{i} + 6\hat{j} - 6\hat{k}$ **(May – 11)**

Vector differentiation

(A) Gradient of a scalar point function ϕ.

1) If $\phi = 3x^2y - y^3z^2$ find $\nabla\phi$ at the point $p(1, -2, -1)$

2) Find $\nabla\phi$, when $\phi = xyz$ at (1, 2, 3). **Ans:** $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

3) (i) If $\phi = x^n + y^n + z^n$ show that $r \cdot \nabla\phi = n\phi$

(ii) If $\phi = x^3 + y^3 + z^3 - 3xyz$, show that $\bar{r} \cdot \nabla\phi = 3\phi$

4) If $\phi = \log\left(x^2 + y^2 + z^2\right)$, find $\nabla\phi$ at (2, 1, 1). **Ans:-** $\dfrac{-\hat{i} + 3\hat{j} + 2\hat{k}}{\sqrt{14}}$

5) If $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$ then show that $\nabla u, \nabla v, \nabla w$ are co-planar.

6) If $\phi = x^2 + y^2 + z^2, \psi = x^2y^2 + y^2z^2 + z^2x^2$ then find $\nabla(\nabla\phi\ \nabla\psi)$

7) If ϕ is a function of u, v, w where u, v, w are functions of x, y, z then

$$\nabla\phi = \frac{\partial\phi}{\partial u}\nabla u + \frac{\partial\phi}{\partial v}\nabla v + \frac{\partial\phi}{\partial w}\nabla w$$

8) Prove that $\nabla f(r) = \dfrac{f'(r)}{r}\bar{r}$ & hence find 'f' if $\nabla f = 2r^4\bar{r}$ **(May – 08)**

9) Prove that :

(i) $\nabla r = \dfrac{\bar{r}}{r}$

(ii) $\nabla \log|\bar{r}| = \dfrac{\bar{r}}{r^2}$

(iii) $\nabla r^n = nr^{n-2}\bar{r}$

(iv) $\nabla\int r^n dr = r^{n-1}\bar{r}$

(v) $\nabla \cdot \bar{r} = 3$

10) Prove that $\nabla\left(r^2e^r\right) = (r + 2)e^r\bar{r}$

11) Prove that :

(i) $\nabla(\bar{a} \cdot \bar{r}) = \bar{a}$

(ii) $(\bar{a} \cdot \nabla)\bar{r} = \bar{a}$

(iii) $(\bar{a} \cdot \nabla)\phi = \bar{a} \cdot \nabla\phi$ where $\bar{a}$ is a instant vector.

12) Prove that :

(i) $\nabla f(u) = f'(u)\nabla u$

(ii) $\nabla\int f(u)du = f(u)\nabla u$

13) Prove that $\bar{a} \cdot \nabla\dfrac{1}{r} = -\dfrac{\bar{a} \cdot \bar{r}}{r^3}$ where $\bar{a}$ is a constant vector.

14) Prove that $\nabla\left[\bar{r}\,\bar{a}\bar{b}\right] = \bar{a} \times \bar{b}$ where $\bar{a}$ and $\bar{b}$ are constant vectors.

15) Prove that $[grad\ f(r)] \times \bar{r} = 0$

16) If $\phi\ \log\left(x^2 + y^2\right)^{1/2}$, prove that grad $\phi = \dfrac{\bar{r} - (\hat{k} \cdot \bar{r})\hat{k}}{\left[\bar{r} - (\hat{k} \cdot \hat{r})\hat{k}\right] \cdot \left[\bar{r} - (\hat{k} \cdot r)\hat{k}\right]}$

17) If $\nabla u = 2r^4\bar{r}$, find u

18) Find $\phi(r)$ such that $\nabla\phi = \dfrac{\bar{r}}{r^5}$ and $\phi(1) = 0$ **Ans :-** $\phi(r) = \dfrac{1}{3} - \dfrac{1}{3r^2}$ **(Dec – 10)**

19) Prove that $\nabla\left[\bar{a} \cdot \nabla\dfrac{1}{r}\right] = \dfrac{3(\bar{a} \cdot \bar{r})\bar{r}}{r^5} - \dfrac{\bar{a}}{r^3}$ where $\bar{a}$ is a constant vector.

20) Prove that $\nabla\left(\dfrac{\bar{a} \cdot \bar{r}}{r^n}\right) = \dfrac{\bar{a}}{r^n} - \dfrac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$ where $\bar{a}$ is a constant vector.

(B)

1) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point (2, -2, 2)

Ans : $\frac{1}{\sqrt{3}}\left(-\hat{i} + \hat{j} + \hat{k}\right)$

2) Find the unit normal to the surface $x^2 + y^2 + z^2 = a^2$ at $\left(\frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right)$

Ans : $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

3) Find the unit normal to the surface $xy^3z^2 = 4$ at the point (-1, -1, 2) **(Dec – 08)**

Ans : $\frac{1}{\sqrt{11}}\left(-\hat{i} + 3\hat{j} - \hat{k}\right)$

4) Find a unit normal to the surface $x^3 + y^3 + 3xy = 3$ at (1, 2, -1)

Ans : $\frac{1}{\sqrt{14}}\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$

(C) Angle between the surfaces

1) What is the angle between the normal to the surface $xy = z^2$ at the points (1, 4, 2) and (-3, -3, 3) ?

Ans : $\theta = \cos^{-1}\left[\frac{1}{\sqrt{22}}\right]$

2) Find the angle between the surface $ax^2 + y^2 + z^2 - xy = 1$ and $bx^2y + y^2z + z = 1$ at p(1, 1, 0).

Ans : 45°

3) Find the cosine of the angle between the normal to the surfaces $x^2y + z = 3$ and $x\log z - y^2 = -4$ at the point of intersection P (–1, 2, 1)

Ans : $\cos\theta = \frac{-5}{\sqrt{18}\sqrt{17}}$

4) Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2).

Ans : $a = 5/2$ & $b = 1$

5) Find the constants m and n such that the surface $mx^2 - 2nyz = (m + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2).

Ans : $m = 5$ & $n = 1$.

6) Find the angle between the two surfaces $x^2 + y^2 + az^2 = 6$ and $z = 4 - y^2 + bxy$ at p (1, 1, 2)

Ans: $\theta = \cos^1\left[\sqrt{\frac{6}{11}}\right]$

7) Find the angle between surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1, -2, 1)

Ans :- $\theta = \cos^{-1}\left(\frac{-3}{7\sqrt{6}}\right)$

8) If the angle between the surfaces $x^2 + axz + byz = 2$ and $x^2z + xy + y + 1 = z$ at (0, 1, 2) is $\cos^1\left(1/\sqrt{3}\right)$, find the constants '$a$' and '$b$'.

9) Find the constants '*a*' *b* & *c* if the normal to the surface $ax^2 + bxz + z^2y = c$ at *P* (-1, 1, 2) is parallel to the normal to the surface $x^2 - y^2 + 2z = 2$ at *Q* (1, 1, 1).

Ans : *a* = 10, *b* = 8 & *c* = -2

10) Find the constants '*a*' and '*b*' if the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at *P* (1, 2, 1).

Ans : *a* = -5, *b* = -7.

11) Find the constants *a* and *b* such that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2y + bz^3 = 4$ cut orthogonally at (1, –1, 2)

Ans : *a* = 4 and *b* = 1

12) Find the constant, *a*, *b*, *c* if the normal to the surface $ax^2 + yz + bxz^3 = c$ at P (1, 2, 1) is parallel to the normal to the surface $y^2 + xz = 61$ at Q (10, 1,6). **(Dec – 10)**

Ans : *a* = 1, *b* = 1, *c* = 4

(D) Directional Derivatives

1) What is the directional derivative of $\phi = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

Ans : $\frac{-11}{3}$

2) Find directional derivative of $\phi = x^4 + y^4 + z^4$ at the point *A*(1, -2, 1) in the direction of line *AB* where *B* = (2, 6, -1) . **(May – 09)**

Ans : $\frac{-260}{\sqrt{69}}$

3) Find the directional derivative of $\phi = e^{2x - y + z}$ at the point (1, 1, -1) in the direction towards the point (-3, 5, 6).

Ans : $\frac{-5}{9}$

4) What is the directional derivative of $\phi = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of the normal to the surface $x \log z - y^2 = -4$ at (–1, 2, 1).

Ans : $\frac{15}{\sqrt{17}}$

5) For the function $\phi = \frac{y}{x^2 + y^2}$ find the magnitude of the directional derivative making an angle of 30° with positive *x*-axis at the point (0, 1).

Ans : $-\frac{1}{2}$

6) Find the directional derivative of $x^3 + y^3 + z^3 - xyz$ at *P* (1,1,1) in the direction of normal to the surface $x \log z + y^2 = 4$ at *Q* (1, -2, 1)

Ans : $\frac{-6}{\sqrt{17}}$

7) Find the rate of change of the distance of $\phi = xyz$ at (1,1,1) in the direction normal to the surface $x^2yz + 4xz^2 = 6$ at the point (1, -2, -1) .

Ans : $\frac{-3}{\sqrt{165}}$

8) Find the directional derivative of $\phi = xy(x - y + 3)$ at P (1, 2, 1) in the direction of normal to the surface $x^2 + y^2 + az^2 = 4$ at Q (1, 1, 1).

Ans : $\sqrt{57}$

9) Find the directional derivative of $\phi = x^2y + y^2z + z^2x^2$ at P (1, 2, 1) in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at a (1, 1, 1).

Ans : $-4\hat{i} + 4\hat{j} + 12\hat{k}$

10) Find the directional derivation of $\phi = e^{2x}\cos yz$ at the origin in the direction of tangent to the curve x = a sin t , y = a cos t , z = at $t = \frac{\pi}{4}$.

Ans : 1.

11) Find the directional derivative of $\phi = x^2 + y^2 + z^2$ at (1, 2, 3) in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ **(Dec – 07)**

Ans : $\frac{52}{\sqrt{50}}$

Divergence

1) $\overline{A} = x^2z\hat{i} - 2y^3z^2\hat{j} + xy^2z\hat{k}$ Find $\nabla \cdot \overline{A}$ at (1, -1, 3).

Ans :- – 47.

2) Given $\phi = 2x^3y^2z^4$. Find $\nabla \cdot \nabla\phi$.

Ans :- $12xy^2z^4 + 2x^3z^4 + 24x^3y^2z^2$

3) Show that $\nabla \cdot \nabla\phi = \nabla^2$ where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ denotes the Laplace's operator.

4) Prove that $\nabla^2(xy + yz + zx) = 0$

5) Prove that $\nabla \cdot \bar{r} = 3$

6) Prove that :

(i) $\nabla \cdot (\bar{a} \times \bar{r}) = 0$ **(May – 08)**

(ii) $\nabla \cdot \left(\frac{\bar{a} \times \bar{r}}{r}\right) = 0$

(iii) $\nabla \cdot (\bar{a} \cdot \bar{r})\,\bar{a} = a^2$ **(May – 08)**

(iv) $\nabla \cdot (\bar{a} \times \bar{r} \times \bar{a}) = 2a^2$

7) Prove that :

(i) $\nabla \cdot \frac{\bar{r}}{r} = \frac{2}{r}$

(ii) $\nabla \cdot \left(\frac{\bar{r}}{r^n}\right) = \frac{(3 - n)}{r^n}$

8) Prove that $\nabla \cdot (r^m\bar{r}) = (m + 3)r^m$

9) Prove that $\nabla\left[\nabla\cdot\frac{\bar{r}}{r}\right]=-\frac{2}{r^3}\bar{r}$

10) Prove that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$ hence find such $f(r)$ that $\nabla^2 f(r)=0$.

11) Prove that :

(i) $\nabla^2\frac{1}{r}=0$

(ii) If $\bar{F}=\frac{x}{r}\hat{i}+\frac{y}{r}\hat{j}+\frac{z}{r}\hat{k}$ and $r=\sqrt{x^2+y^2+z^2}$ prove that $\nabla\cdot\bar{F}=\frac{2}{r}$.

12) Prove that :

(i) $\nabla^2\left(r^2\log r\right)=5+6\log r$

(ii) $\nabla^2\left(r^n\log r\right)=\left[n(n+1)\log r+2n+1\right]r^{n-2}$

(iii) $\nabla^2\left(\frac{1}{r^2}\right)=\frac{2}{r^4}$ (iv) $\nabla^2(r^2)=6$

13) Prove that $\nabla^2\left(r^2e^r\right)=\left(r^2+6r+6\right)e^r$

14) Prove that $\nabla^4\left(r^2\log r\right)=\frac{6}{r^2}$

15) Prove that $\nabla^4 e^r=\left[1+\frac{4}{r}\right]e^r$

16) (i) Prove that $\nabla\cdot\left[r\nabla\cdot\left(\frac{1}{r^3}\right)\right]=\frac{3}{r^4}$ (ii) Prove that $\nabla\cdot\left(r\ \nabla\frac{1}{r^n}\right)=\frac{n(n-2)}{r^{n+1}}$ **(Dec – 09)**

17) If ∇r^n is solenoidal, show that $n = -1$.

18) If $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$ & $\bar{a}\cdot\bar{b}$ are constant vectors, prove that

$\bar{a}\cdot\nabla\left(\bar{b}\cdot\nabla\frac{1}{r}\right)=3\frac{(\bar{a}\cdot\bar{r})(\bar{b}\cdot\bar{r})}{r^5}-\frac{\bar{a}\cdot\bar{b}}{r^3}$ **(Dec – 07)**

Curl :

1) If $A=x^2z\hat{i}-2y^3z^2\hat{j}+xy^2z\hat{k}$. Find $\nabla\times\bar{A}$ at (1, -1, 1)

Ans.: $-6\hat{i}$

2) Prove that :

(i) Curl grad $\phi = 0$

(ii) Div curl $\bar{A} = 0$

(iii) Curl $\bar{r} = 0$

(iv) Curl $\left(r^m\bar{r}\right)=0$.

3) Prove that $\nabla\times(\bar{a}\times\bar{r})=2\bar{a}$ where $\bar{a}$ is a constant vector. **(May – 08)**

4) Prove that $\nabla\times(\bar{a}\times\nabla\log r)=\frac{2(\bar{a}\cdot\bar{r})\bar{r}}{r^4}$

5) Prove that $\nabla\times\left(\frac{\bar{a}\times\bar{r}}{r^n}\right)=\frac{(2-n)}{r^n}\bar{a}+\frac{n(\bar{a}\cdot\bar{r})\bar{r}}{r^{n+2}}$ **(May – 09)**

6) Prove that $\nabla\times\left(\frac{\bar{a}\times\bar{r}}{r^3}\right)+\nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^3}\right)=0$

7) Prove that $\nabla\times\left(\frac{\bar{a}\times\bar{r}}{r}\right)=\frac{\bar{a}}{r}+\frac{\bar{a}\cdot\bar{r}}{r^3}\bar{r}$

8) Find '*n*' such that $\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^n} \right) = \frac{2(\bar{a} \cdot r)}{r^4} \bar{r}$

9) Find f(r) so that the vector $f(r)\bar{r}$ is both solenoidal and irrotational. **(Dec – 09)**

Ans : c / r^3

10) If $\bar{a}$ is a constant vector, prove that $\nabla \times [\bar{r} \times (\bar{a} \times \bar{r})] = 3(\bar{r} \times \bar{a})$.

11) If $u = x^2 + y^2 + z^2$ and $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then find $div(u\bar{r})$ in terms of *u*.

12) Find the value of '*n*' for which the vector $r^n \bar{r}$ is solenoidal, where $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Ans :- $n = -3$.

13) Prove that $\bar{F} = (x + 2y + az)\hat{i} + (bx - 3y - 3)\hat{j} + (4x + cy + 2z)\hat{k}$ is solenoidal and determine constants *a*, *b* & *c* if $\bar{F}$ is rotational.

Ans :- $a = 4$, $b = 2$, $c = -1$.

14) Prove that $\nabla \cdot \left[\frac{f(r)}{r} \bar{r} \right] = \frac{1}{r^2} \frac{d}{dr} \left[r^2 f(r) \right]$.

Hence, or otherwise prove that $div\left(r^n \bar{r}\right) = (n+3) r^n$ **(Dec – 08)**

15) Verify : $\left(\bar{a} \times \bar{b}\right) \times \bar{c} = (\bar{a} \cdot \bar{c})\bar{b} - \left(\bar{b} \cdot \bar{c}\right)\bar{a}$ and $\bar{a} \times \left(\bar{b} \times \bar{c}\right) = (\bar{a} \cdot \bar{c})\bar{b} - \left(\bar{a} \cdot \bar{b}\right)\bar{c}$.

For $\bar{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$, $\bar{b} = 6\hat{i} + 4\hat{j} + 2\hat{k}$, $\bar{c} = 3\hat{i} + 2\hat{j} + 4\hat{k}$ **(May – 10)**

16) Find $\nabla \cdot \bar{F}$ and $\nabla \times \bar{F}$ where $\bar{F} = \frac{\left(x\hat{i} - y\hat{j}\right)}{\left(x^2 + y^2\right)}$ **(Dec – 10)**

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Analytic Functions

Problem

01. Show that the functions z^2, sin z , cosh z **(M-10)**, log z and ze^{-z} are analytic.

02. Show that the following functions are analytic

(a) $(x^3 - 3xy^2) + i(3x^2y - y^3)$ (b) $\frac{x - iy}{x^2 + y^2}$ (c) sin x cosh y + i cos x sinh y

03. Show that the following functions satisfy Cauchy-Riemann equations at the origin but are not analytic at the origin.

(a) $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, z \neq 0 \\ 0 \qquad\qquad\qquad , z = 0 \end{cases}$ (b) $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2 + y^4}, z \neq 0 \\ 0 \qquad\qquad , z = 0 \end{cases}$

(c) $f(z) = |z|^2$ (d) $f(z) = \begin{cases} e^{-z^4}, z \neq 0 \\ 0 \quad , z = 0 \end{cases}$

Sufficient Conditions

04. Show that the functions $\bar{z}$ **(M-11)** & $\frac{z}{\bar{z}}$ are not analytic.

05. Find the constant a in the analytic function $\frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{ay}{x}$

Harmonic Functions

06. S.T. $u = x^2 - y^2$ & $v = \frac{-y}{x^2 + y^2}$ are harmonic s but u + iv is not analytic **(M-11,D-08)**

07. Prove that there does not exist any analytic function whose real part is $x^2 + 3x + y^2 - 4y + 6$

Polar Form

08. *If* $f(z) = (r^3 \cos k\theta + ir^k \sin k\theta)$ is analytic find k **(D-09)**

09. Show that $u = (r - \frac{a^2}{r})\sin\theta$ is harmonic.

Milne-Thomson Method

10. S.T. the following functions are harmonic and find the harmonic conjugate of the following functions & the corresponding analytic function $f(z) = u + iv$ in terms of z

(a) $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 2x + 1$ (b) $u = e^{-2xy}\sin(x^2 - y^2)$ **(D-10)**

$(c)\ v = \dfrac{\sinh 2y}{\cosh 2y + \cos 2x}$ **(D-08)** $(d)\ v = (r - \frac{a^2}{r})\sin\theta$

Orthogonal Family of Curves

12. Find the family of curve orthogonal to

(a) $e^{-x}(x\sin y - y\cos y) = c$ **(D-08)**

(b) $3x^2y + 2x^2 - y^3 - 2y^2 = c$

(c) $x^3y - xy^3 = c$ **(M-09)**

13. Find the analytic function f(z)=u+iv ,in terms of z, if

(a) $u - v = e^x(\cos y - \sin y)$ **(M-08)**

(b) $u - v = (x - y)(x^2 + 4xy + y^2)$ **(D-09)**

(c) $\frac{u}{v} = \tan y$

14. If f(z)=u+ iv is analytic then show that it is a constant function if

(a) $\overline{f(z)}$ is analytic

(b) f(z)(has constant modulus

(c) f(z) has constant amplitude

15. If f(z)=u+ iv is analytic then show that

(a) $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ **(M-10)**

(b) $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u^2 = 2|f'(z)|^2$

(c) $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$ **(D-10)**

16. Show that $V = e^{-x}(x\cos y + y\sin y)$ is harmonic and the corresponding analytic function f(z)=u + iv **(M-09)**

17. Find the analytic function f(z) whose real part is $(r^2\cos 2\theta - r\sin\theta)$

18. Show that $f(z) = (x^3 - 3xy^2 + 2xy) + i(3x^2y - x^2 + y^2 - y^3)$ is analytic and hence find $f'(z)$ **(M-09)**

18. Show that the following functions are analytic

(a) $x^2 - y^2 + i2xy$ (b) $e^x(\cos y + i\sin y)$

19. Find a, b, c, d if $(x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$ is analytic. **(D-08)**

20. Find the value of a if $u = x^2 - y^2$ is a harmonic & its harmonic conjugate **(M-11)**

21. Show that $u = e^x\cos y + x^3 - 3xy^2$ is harmonic **(M-08)**

22. If ϕ and ψ are harmonic functions in x and y then show that u + iv is analytic

Where $u = \frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial x}$ and $v = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial y}$ **(M-09)**

23. Find the family of curves orthogonal to $e^{-x}\cos y + xy = c$

24. Find the harmonic conjugate of the following functions and the corresponding analytic function $f(z) = u + iv$ in terms of z

(a) $v = 3x^2y + 6xy - y^3$ (b) $v = 2xy - \frac{y}{x^2 + y^2}$ (c) $u = (r + \frac{a^2}{r})\cos\theta$

25. Find the analytic function $f(z) = u + iv$, in terms of z , if

(a) $u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ (b) $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$ **(D-10)**

26. If f(z)=u+ iv is analytic then show that it is a constant function if

(a)v+ iu is analytic (b)u-iv is analytic

27. If f(z)=u+ iv is analytic then S.T. $\left(\frac{\partial}{\partial x}|f(z)|^2\right) + \left(\frac{\partial}{\partial y}|f(z)|^2\right) = |f'(z)|^2$

Conformal Mapping

Problem

1. If w=f(z) is analytic and $f'(z) = 0$ in a region R then prove that w=f(z) is conformal in that region.

2 .Find image of circle lzl=2under

(i)w=z+(3+2i) (ii)w=3z (iii)w= $\sqrt{2}e^{\frac{i\Pi}{4}}z$ (iv)w=(1+2i)z+(3+4i) (v)w=1/z

3. Show that under the transformation w= 1/z
   (a) circles in the z-plane are mapped into circles in the w-plane
   (b)circle $|z-1|=1$ is mapped into the line u=1/2
   (c) circle $|z-3|=5$ is mapped into the line $\left|w+\frac{3}{16}\right|=\frac{5}{16}$
   (d) line y-x+1=0 is mapped into the circle $u^2+v^2-u-v=0$
   (e) hyperbola $x^2-y^2=1$ is mapped into the lemnicate $\rho^2=\cos 2\phi$

4. Under the transformation $w=z^2$ find the image of
   (a) the region between the lines x=0, x=1, y=0 and y=1
   (b) the region between the lines x=1, y=1, x=2 and y=2
   (c) the region between the lines x=1, y=1 and x+y=1

5. Show that the transformation
   (a) $w=z+\frac{a^2-b^2}{4z}$ maps the circle $|z|=\frac{a+b}{2}$ in the z-plane into an ellipse in the w-plane with semi-axes a and b.
   (b) $w=z+\frac{a^2}{z}$ maps the circle $|z|=b$ in the z-plane into an ellipse when b>a and into a line when b<a
   (c) $w=z+\frac{1}{z}$ maps lines amp z=constant($<\frac{\pi}{2}$)and circles $|z|=$constant into confocal conics with foci at w= $\pm$ 2i

Bilinear transformation

6. Prove that a bilinear transformation
   (a)is a combination of translation, magnification, rotation & inversion; hence deduce that it maps circles in the z-plane into circles in the w- plane.
   (b) keeps the cross-ratio invariant

7. Show that the bilinear transformation

(a) $w=\frac{5-4z}{4z-2}$ maps the circle $|z|=1$ into the circle $\left|w+\frac{1}{2}\right|=1$

(b) $w=\frac{2z+3}{z-4}$ maps the circle $|z-2|=2$ into the line 4u+3=0

8. Find a bilinear transformation which maps
   (a) z=1, i,-1 into w=I, 0, -i and hence find the image of the region IzI<1 **(M-09)**
   (b) z=-1, 1, ∞ into w= -i,-1, i and hence find the fixed points **(M-11)**

9. Find image of the triangular whose vertices are i, 1+i, 1-i under the transformation w= 3z+4-2i **(D-10)**

10. Find image of the rectangle bounded by the lines x=0,y=0,x=2,y=2 under the transformations
   (a) w=(1+i)z (b) w=(1+2i)z+(1+i) (c) w=z+(1+i)

11. Show that under the transformation $w=\frac{1}{z}$
   (a) the circle $|z-2i|=2$ is mapped into the line v=-1/4 **(M-09)**
   (b) the circle $(x-3)^2+y^2=2$ is mapped into the circle $(u-\frac{3}{7})^2+v^2=\frac{2}{49}$
   (c) the strip $\frac{1}{4}\le y\le\frac{1}{2}$ is mapped into the region between the two circles $u^2+(v+2)^2=4$ and $u^2+(v+1)^2=1$

12. Show that under the transformation $w=\frac{iz+2}{4z+i}$ maps the real axis in the z-plane into a circle in the w-plane.Find the centre and radius of the circle .Find the point in the z-plane which is mapped onto the centre of the circle in the w-plane .

13. Under the transformation $w=z^2$ find the image of
   (a) the region between $0\le r\le 1$ and $0\le\theta\le\frac{\pi}{4}$
   (b) the circle $|z-1|=1$ as the car diode $\rho=2(1+\cos\phi)$ **(M-09)**

14. Show that the transformation
   (a) $w=z^2+z$ maps the circle $|z|=1$ in the z-plane into the car diode **(D-08)** $\rho=2(1+\cos\phi)$ in the w-plane
   (b) $w=\sin z$ maps lines parallel to the co-ordinate axes in the z-plane into confocal conics in the w-plane

(c) w= cosh z maps lines x=constant and y=constant into confocal conics in the w-plane

(d) $w = z + \frac{1}{z}$ maps lines amp z=constant($< \frac{\pi}{2}$)and circles $|z| = $ constant into confocal conics with foci at w= $\pm 2i$

15. Prove that a bilinear transformation can be expressed as

(a) $\frac{1}{w-\alpha} = k + \frac{1}{z-\alpha}$ if α is a single fixed point

(b) $\frac{w-\alpha}{w-\beta} = k\frac{z-\alpha}{z-\beta}$ if α and β are two fixed points

16 . Show that the bilinear transformation

(a) $w = \frac{3-z}{z-2}$ maps the circle in the z-plane with center (5/2,0) and radius ½ into the imaginary axis in the w-plane

(b) $w = \frac{2}{z+i}$ maps the real axis in the z-plane into a circle in the w-plane.

17. Find a bilinear transformation which maps

(a) z=2, i, -2 into w=1, i, -1 **(M-10,D-09)**

(b)z= 0, i, -2i into w=-4i, ∞,0 **(D-10,D-08,M-08)**

And find the fixed point of the transformation

18. Find the fixed points and normal form of the bilinear transformation

(a) w= $\frac{3z-4}{z-1}$ (b) $w = \frac{z-4}{2z-5}$

BESSEL FUNCTIONS

01) Reduce Laplace's equation in Cartesian co-ordinates to Bessel's equation by changing to cylindrical co-ordinates.

02) Prove that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ **(M-09)** and $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$

03) Show that (a) $y = x^{-n}J_n(x)$ is a solution of $x\frac{d^2y}{dx^2}+(2n+1)\frac{dy}{dx}+xy=0$

(b) $y = xJ_n(x)$ is a solution of $x^2\frac{d^2y}{dx^2}-xy'+(x^2-n^2+1)y=0$

04) When n is a positive integer prove that $J_n(-x)=(-1)^nJ_n(x)$ and deduce that $J_n(x)$ is even function when n is even & an odd function when n is odd.

05) Find $J_0(x)$ and $J_1(x)$

08) When n is a positive integer prove that $J_{-n}(x)=(-1)^nJ_n(x)$

Recurrence Relations

Problems

11) Find $J_2(x), J_3(x), J_4(x), J_5(x)$ in terms of $J_0(x)$ *and* $J_1(x)$

12) P.T. $J_{3/2}(x)=-\sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x}-\cos x\right]$ & $J_{5/2}(x)=\sqrt{\frac{2}{\pi x}}\left[\frac{(3-x^2)\sin x-3x\cos x}{x^2}\right]$

13) Show that (a) $\frac{d}{dx}(J^2n+J^2n+1)=2\left(\frac{n}{x}J_n^2-\frac{n+1}{x}J_{n+1}^2\right)$

(b) $\frac{d}{dx}(xJ_nJ_{n+1})=x\left(J^2n-J^2n+1\right)$

14) S.T. (a) $2J_n'=J_{n-1}-J_{n+1}$

(b) $2^2J_n''=J_{n-2}-2J_n+J_{n+2}$ (M-09) *and hence* $J_2-J_0=2J_0''$

(c) $2^3J_n'''=J_{n-3}-3J_{n-1}+3J_{n+1}-J_{n+3}$ *and hence* $3J_1-J_3=4J_0'''$

15) Show that (a) $J_0'(x) = -J_1(x)$

(b) $J_2'(x) = \frac{2}{x}J_0(x) + \left(1 - \frac{4}{x^2}\right)J_1(x)$

16) Show that (a) $\int_0^1 x^{5/2} J_{3/2}(ax)dx = \frac{1}{a}J_{\frac{5}{2}}(a)$ **(D-10)**

(b) $\int x^3 J_3(x)dx = -x^3 J_2(x) - 5x^2 J_1(x) - 15xJ_0(x) + 15\int J_0(x)dx$

(c) $\int x^4 J_1(x)dx = (4x^3 - 16x)J_1(x) - (x^4 - 8x^3)J_0(x) + C$

(d) $\int J_3(x)dx = -2\frac{J_1(x)}{x} - J_2(x)$ **(D-10)**

17) Prove that (a) $J_{-3/2}(x) = -\sqrt{\frac{2}{\pi x}}\left[\frac{\cos x}{x} + \sin x\right]$ **(D-09)**

(b) $J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{(3 - x^2)\cos x + 3x\sin x}{x^2}\right]$

(c) $J_4(x) = \left(1 - \frac{24}{x^2}\right)J_0(x) + \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x)$

(d) $J_3'(x) = \left(\frac{12}{x^2} - 1\right)J_0(x) + \left(\frac{5}{x} - \frac{24}{x^3}\right)J_1(x)$

(e) $\int x^4 J_0(x)dx = x^2(x^2 - 9)J_1(x) + 3x(x^2 - 3)J_0(x) + 9\int J_0(x)dx$

Generating Function for $J_n(x)$

Problems

18) Show that (a) $\cos(x\sin\theta) == J_0(x) + 2\cos\theta J_2(x) + 2\cos 4\theta J_4(x) + ...$

(b) $\sin(x\sin\theta) == 2\sin\theta J_1(x) + 2\sin 3\theta J_3(x) + 2\sin 5\theta J_5(x) + ...$

(c) $\cos x = J_0(x) - 2J_2(x) + 2J_4(x) + ...$

(d) $\sin x = 2J_1(x) - 2J_3(x) + 2J_5(x) + ...$

(e) $x == 2\{J_1(x) + 3J_3(x) + 5J_5(x) + ...\}$

(f) $J_0^2 + 2J_1^2 + 2J_2^2(x) + ... = 1$

19) Prove that the Bessel's integral

(a) $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\theta - x\sin\theta)d\theta$

(b) $J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\sin\theta)d\theta$

Fourier-Bessel Series

Problems

20) (a)Expand $f(x) = 4x - x^3$ in 0<x<2 as $4x - x^3 = 8\sum \frac{J_2(2\lambda_n)}{\lambda_n{}^2 J_2{}^2(2\lambda_n)} J_1(\lambda_n x)$

where λ_n's are the positive roots of $J_1(2\lambda) = 0$ **(D-10)**

(b) Show that the Fourier- Bessel series in $J_2(\lambda_i x)$ for $f(x) = x^2$ in 0<x<a

where λ_i are the positive roots of $J_2(\lambda a) = 0$ is $x^2 = \sum_{i=1}^{\infty} \frac{2a}{\lambda_i J_3(\lambda_i a)} J_2(\lambda_i x)$

21) (a)Expand f(x)=1 in 0<x<1 in a series as $1 = \sum_{i=1}^{\infty} \frac{2}{\lambda_i J_1(\lambda_i)} J_0(\lambda_i x)$ **(D-09)**

(b) Show that $x = \sum_{i=1}^{\infty} \frac{2}{\lambda_i J_2(2\lambda_i)} J_1(\lambda_i x)$

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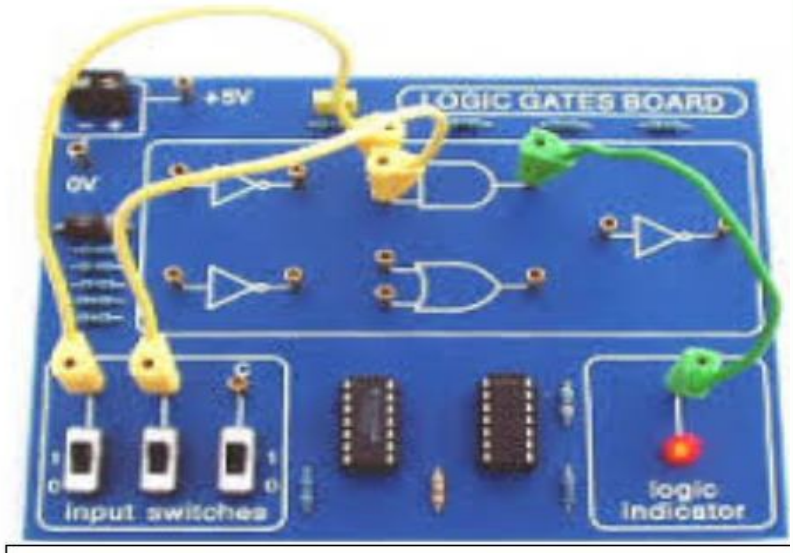
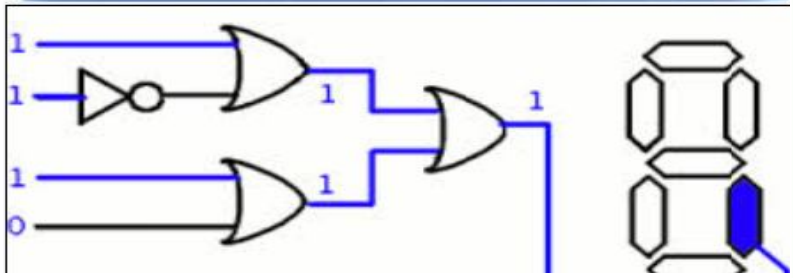




SH 2015

DIGITAL CIRCUITS AND DESIGN

Mrs. LEENA GOVEKAR

GROUP NAME : DIGITAL ELECTRONICS	
COURSE TITLE: Digital Circuits and Design	COURSE CODE: EXE 303 SEM :-III

Subject Plan

TEACHING AND EXAMINATION SCHEME:

TEACHING SCHEME		EXAMINATION SCHEME				
Theory	Practical's	Paper (Hrs)	Theory	Practicals Orals	Term Work	Total
04	02		04	50	25	75

Course Objectives:

1. To deliver the knowledge, motivate and train students in logic design

2. To introduce the students to various logic gates, SOP, POS and their minimization techniques.

3. To explain and describe various logic families and to provide information on different IC's.

4. To teach the working of combinational circuits and their applications.

5. To make students aware of characteristics of various types of SSI, LSI and MSI devices and their use in various applications.

6. To teach students to analyze, understand and design sequential circuits.

7. To describe State Machines and explain their design using state diagrams.

8. To explain various types of programmable devices

9. To train students in writing program with hardware description languages.

10. To prepare students for understanding courses like microprocessors, microcontrollers, VLSI design, embedded systems and digital communications

Course Outcome:

1. Ability to develop a logic and apply it to solve real life problems

2. Ability to understand current applications, trends and new directions in logic design

3. Ability to reduce SOP and POS equations.

4. Ability to understand differences between logic families TTL and CMOS

5. Ability to understand various SSI, LSI and MSI devices

6. Ability to use SSI, LSI and MSI devices in various applications

7. Ability to analyze, design and implement combinational circuits

8. Ability to analyze, design and implement sequential circuits

9. Ability to solve state machines

10. Ability to design state machines using state diagrams, State Reduction techniques and State machine synthesis using transition lists

11. Ability to understand the concept of simulation, synthesis and implementation

12. Ability to use hardware description languages for logic circuit design.

13. Ability to understand programmable logic devices

14. Ability to program CPLD and FPGA

Subject Title: Digital Circuits and Design	
Chapter No. : 1	**Approximate Time Needed :14 H**
Chapter Name:	**Fundamentals of Digital Design**

Chapterwise Plan

Lesson Schedule:-oriented programming : Classes, Objects, Messages,

1. **Logic gates –Review of basic gates.**
2. **Universal gates.**
3. **Sum of product and product of sum.**
4. **Minimization with K map.**
5. **Realization using mixed logic and universal logic.**
6. **Logic families-Types of logic families, characteristic parameters.**
7. **Transfer characteristic of TTL NANAD.**
8. **Interfacing CMOS to TTL and TTL to CMOS.**
9. **Combinational circuits using basic gates as well as MSI devices.**
10. **Half adder, full adder ,Half subtracted ,Full subtracted.**
11. **Multiplexer, DE multiplexer ,decoder**
12. **7483.**
13. **74151**
14. **7485.**

Objective: The Student will learn fundamental concepts of Digital Electronics.

Model Questions:

4. **What are advantages of Digital logic**
5. **Compare different logic families.**
6. **Implement 8 bit comparator using 7485.**
7. **Design full adder using suitable multiplexer.**
8. **Minimize using k-map sop equation f(A,B,C,D)= (2,3,7,8,10,12,15)**
9. **Implement BCD adder using 7483**

Subject Title: Digital Circuits and Design	
Chapter No. : 2	**Approximate Time Needed :10 H**
Chapter Name:	**Elements of Sequential Logic**

Chapter wise Plan

Lesson Schedule :

Class No Portion covered per hour

1. **Latches and flip-flops.**
2. **Types of flip flops.**
3. **Truth table,Excitation table of all flip flops.**
4. **Application of flip flops.**
5. **Asynchronous counters.**
6. **Synchronous counters.**
7. **Mod counters.**
8. **Shift Registers.**
9. **Ring ,twisted ring counters.**
10. **Universal shift Register.**

Objective: The Student will learn fundamental concepts of Java programming.

Model Question:-

1. **Describe all types of flip-flops.**
2. **Design Mod 5 Asynchronous counter.**
3. **Design Mod 12 synchronous counter using JK flip flop.**
4. **Draw Universal Shift Register.**
5. **Design Up down counter.**

Subject Title: Digital Circuits and Design	
Chapter No. : 3	**Approximate Time Needed :10 H**
Chapter Name:	**Sequential Logic Design**

Chapter wise Plan

Lesson Schedule :

Class No Portion covered per hour

1. **Mealy and Moore machine.**
2. **Clocked synchronous state machine Analysis.**
3. **Sequence detection.**
4. **State Assignment ,Reduction techniques.**
5. **Clocked synchronous state machine design.**
6. **Design problems.**
7. **MSI IC 7490,74163.**
8. **74169**
9. **74194.**
10. **Applications of IC's.**

Objective: The Student will learn fundamental concepts of sequential machines.

Model Question.

1. **Design Mealy machine for detecting sequence 1010.**
2. **Design MOD 63 counter using 74163IC.**
3. **Design Ring counter using 74194IC.**

Subject Title: Digital Circuits and Design	
Chapter No. : 4	**Approximate Time Needed :07 H**
Chapter Name:	**Programmable Logic Devices**

Chapter wise Plan

Lesson Schedule :
Class No Portion covered per hour

1. **Concepts of PAL and PLA.**
2. **Design problems using PAL.**
3. **PLA.**
4. **Introduction to CPLD.**
5. **Architecture of CPLD.**
6. **Introduction to FPGA.**
7. **Architecture of FPGA.**

Objective: The Student will learn complex programmable logic devices

1 Able to design digital logic using programmable devices.

2 Implement combinational logic using PAL and PLA.

3 Describe CPLD

4 Describe Architecture of FPGA.

Subject Title: Digital Circuits and Design	
Chapter No. : 5	**Approximate Time Needed :07 H**

Chapter wise Plan

Lesson Schedule :

Class No Portion covered per hour

1. **Functional simulation.**
2. **Timing simulation.**
3. **Logic synthesis RTL.**
4. **Introduction to VHDL.**
5. **Modeling styles of VHDL**
6. **VHDL programs**
7. **VHDL programs.**

Objective: The Student will learn VHDL>

Model Question:

1. **Write a program in VHDL for logic gates.**
2. **Write a program in VHDL for combinational circuits**
3. **Write a program in VHDL for sequential circuits.**

Chapter Name:	**Simulation**

Subject Title: Digital Circuits and Design	
Chapter No. : 6	**Approximate Time Needed :06 H**
Chapter Name:	**Testability**

Chapter wise Plan

Lesson Schedule :

Class No Portion covered per hour

1. **Fault models.**
2. **Bridging faults.**
3. **Controllability and observability.**
4. **Path sensitization, ATPG>**
5. **Design for testability.**
6. **Boundary scan logic ,JTAG and built in self-test.**

Objective:-Student will learn about fault models and testability.

Model Questions:-

1. **Write note on faults model and Bridging**
2. **Write short note on boundary scan logic.**
3. **Short note on JTAG**
4. **Built in self test.**

25-11-2013-DTP-P-7-RA-10

4|12|13

Con. 9591-13. **GX - 12110**

(3 Hours) **[Total Marks :80**

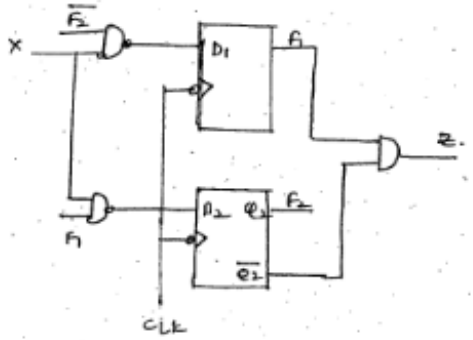
N. B. : (1) Question No. **1** is **compulsory**.
(2) Solve any **three** questions from remaining **five** questions.
(3) Draw neat **diagrams** wherever **necessary.**
(4) Furnish neat **sketches** and assume **suitable data** if required.

1. (a) Explain Moore and Mealy type of sequential circuits. **5**
(b) Draw the circuit diagram of 2-input TTL NAND gate. **5**
(c) Explain the term noise margin and its value for TTL and CMOS family. **5**
(d) Explain stuck at '0' and stuck at '1' faults. **5**

2. (a) Draw the circuit diagram of J-K F/F using NAND gates. Derive it's characteristic equation and excitation table. What is race around condition in J-K F/F and how it is avoided ? **10**
(b) Design and explain 8 bit binary adder using IC 7483. **10**

3. (a) Implement following functions using NAND gate only : **10**
(i) $F = \Sigma m (1, 2, 3, 4, 7, 11, 13) + d (9, 15)$
(ii) $F = \pi M (4, 5, 6, 7, 8, 12) + d (1, 2, 3)$

(b) Analyze the sequential state machine shown in **fig.** obtain the state diagram for the same. **10**



4. (a) Design Moore sequence detector to detect a sequence ------101------ using D F/F. **10**
(b) Discuss XC 4000 FPGA architecture with neat block diagram. **10**

[TURN OVER

5. (a) Construct ring counter using IC 74194 and draw the output waveform. **10**
   (b) Identify indistinguishable states in the following state table and obtain minimized state diagram **10**

PS	NS		OIP	
	X = o	X = 1	X = o	X = 1
1	2	3	0	0
2	2	4	0	0
3	2	3	0	0
4	5	3	0	0
5	2	6	0	1
6	5	3	0	0

6. Write a short notes on any **three** : – **20**
   (a) JTAG and BIST
   (b) VHDL
   (c) PAL AND PLA
   (d) XC 9500 CPLD family.

December 2014

(3 Hours) **[Total Marks : 80**

N.B.: (1) Question No. **1** is **Compulsory.**

(2) Solve any **Three** from remaining **Five** questions.

(3) Draw **neat** logic diagram and assume suitable data wherever necessary.

Q 1 (a) Interfacing between CMOS and TTL **05**

(b) Convert **T** flip-flop to **D** flip-flop **05**

(c) XC 4000 FPGA architecture block diagram **05**

(d) Draw truth table and logic diagram of Full subtractor **05**

Q 2 (a) Write a VHDL code for Full adder **10**

(b) Design MOD 10 asynchronous counter. **10**

Q 3 (a) Design a mealy sequence detector to detect ---1010--- using D flip-flops and logic gates **10**

(b) Design a circuit with optimum utilization of PLA to implement the following functions **10**

$R = \sum m (0, 2, 5, 7, 11, 12)$

$P = \sum m (1, 3, 8, 9, 11, 13)$

$Q = \sum m (0, 5, 8, 12, 14)$

Q 4 (a) Implement following function using 8:1 MUX and logic gates **10**

$P(X,Y,Z,W) = \sum m (0,3,4,7,8,9,13,14)$

(b) Eliminate redundant states and draw reduced state diagram **10**

PS	NS		O/P
	X=0	X=1	Y
A	B	C	1
B	D	C	0
C	F	E	0
D	E	B	1
E	B	C	1
F	C	E	0
G	F	G	0

Q 5 (a) Use K-map to reduce following function and then implement it by NOR gates. **10**

$F = \pi M (0, 3, 4, 5, 8, 10, 12, 14) + d (2, 9)$

(b) Design 8 bit up counter using IC 74163, draw a circuit diagram and explain its working. **10**

6. Write short notes on any **three** **20**

i) Noise Margins

ii) JTAG and BIST

iii) PAL and PLA

iv) Stuck at '0' and '1' faults

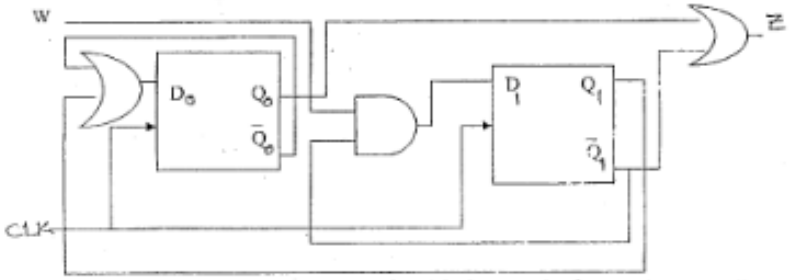
January 2014

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. **1** is **compulsory.**
(2) Solve any **three** from remaining **5** questions.
(3) Draw **neat** diagrams wherever **necessary.**

1. **(A)** Implement the following function using NOR gates only (after reduction using K map) 10
F = π M (1,2,4,7,11,13) . d (9,15)
(B) Design a MOD 6 asynchronous counter and explain glitch problem. 10

2. **(A)** Analyze the clocked synchronous machine given below. Write excitation equations, excitation/transition table and state /output table (Use state names A - D for Q1-Q2=00-11). Also draw the state diagram. 10



(B) Design a 1 digit BCD adder using IC 7483 and explain the operation for $(0111)_{BCD} + (1001)_{BCD}$. 10

3. **(A)** Write a VHDL code for 8:1 Multiplexer with active low enable input. 10
(B) Design a mealy sequence detector to detect a sequence ----1101---using D flip-flops and logic gates. 10

4. **(A)** Design a circuit with optimum utilization of PLA to implement the following functions 10
$F1 = \sum m\ (1, 2, 3, 6, 9, 11)$
$F2 = \sum m\ (0, 1, 6, 8, 9)$
$F3 = \sum m\ (2, 3, 8, 9, 11)$
(B) Implement following function using 4:1 line MUX and NAND gates. 10
$F\ (A, B, C, D) = \sum m\ (1, 2, 6, 7, 10, 12, 13)$

5. **(A)** Design a 8 bit binary up counter using MSI counter IC 74163, draw a circuit diagram and explain working. 10
(B) Eliminate redundant states and draw reduced state diagram. 10

PS	NS		O/P
	X = 0	X = 1	Y
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0

6. Write short notes on (Any THREE): 20
1. XC 4000 FPGA Architecture
2. Stuck at '0' and stuck at '1' fault
3. Master Slave JK flip flop
4. 2 input TTL NAND gate

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Con. 11974-14.

January 2015

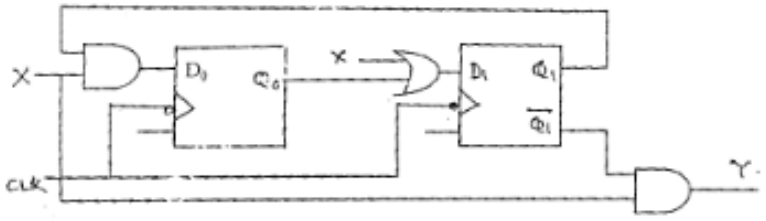
(3 Hours) [Total Marks : 80

N.B.: (1) Question **No.1** is **compulsory**.
(2) Solve any **three** from remaining **five** questions.
(3) Draw **neat** diagram wherever **necessary**.

1. (a) Explain the current sinking and sourcing when two standard TTL gates are connected. 5
   (b) Explain glitch problem of ripple counter along with waveform. 5
   (c) Draw truth table and circuit of JK flip flop using NAND gates. 5
   (d) Draw internal block diagram of IC 7490. 5

2. (a) Design 4 bit ring counter using IC 74194 and draw Its output waveform. 10
   (b) Discuss CPLD XC 9500 architecture with neat block diagram. Describe main Features. 10

3. (a) Design MOD 11synchronous counter using T flip flop. 10
   (b) Identify the circuit shown in figure. Write the state table and draw state diagram for the same. 10



4. (a) Implement 10 bit comparator using IC 7485. 10
   (b) Simplify following logic function and realize using NOR gates. 10
   $f(w,x,y,z) = \pi M(1, 2, 3, 7, 10, 11) + d(0,15)$
   $f(w,x,y,z) = \pi M(3, 4, 5, 6, 7, 10, 11, 15)$

5. (a) Identify indistinguishable state in following state table and obtain minimized state diagram **10**

PS	X = 0		X = 1	
	NS	Output	NS	Output
A	A	0	A	0
B	A	1	F	1
C	D	0	E	0
D	A	1	G	0
E	B	0	C	0
F	D	0	D	0
G	B	0	C	0

(b) Draw a circuit diagram of a CMOS inverter. Draw its transfer Characteristics and explain its operation. **10**

6. Write a short note on (any three) **??**
   (i) K-map.
   (ii) Automatic Test Pattern Generation (ATPG).
   (iii) Mealy and Moore sequential machine.
   (iv) SR flip flop.